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FIFTH CONFERENCE ON NONLINEAR VIBRATIONS, STABILITY, AND DYNAMICS OF STRUCTURES

June 12-16, 1994

Program

Virginia Polytechnic Institute and State University
Partially Sponsored by the Army Research Office

Dedicated to the Memory of P. Sethna and I. Tadjbakhsh

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13. ABSTRACT *(Maximum 200 words)*

This conference was the fifth in a series of meetings focused on several special research topics in the study of the non-linear dynamic response of elastic structures. The meeting was well attended, and participants included representatives from the United States, Canada, Europe, and Japan. The meeting lasted four and a half days, and the agenda consisted of a sequence of single sessions (i.e., there were no parallel sessions). Typically, five basic research papers were presented in each session. The research topics addressed included structural control, parametric vibrations, computational methods, multi-body dynamics, chaos, non-linear normal modes, and structural dynamics.

14. SUBJECT TERMS
Non-linear Vibrations, Multi-body Dynamics, Parametric vibrations, Elastic Stability, Structural Control

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Sunday, June 12

1320-1330 Opening Remarks

Session 1. Nonstationary and Random Vibrations

Chairmen: Gary L. Anderson, Army Research Office, Research Triangle Park, NC and P. S. Symonds, Brown University, Providence, RI

1330-1510

Nonstationary (NS) Food for Thought
R. M. Evan-Iwanowski, University of Central Florida, Orlando, FL, C. H. Lu, Memphis State University, Memphis, TN, and G. L. Ostiguy, Montreal Polytechnical Institute, Montreal, Quebec, CANADA

Nonstationary Transition Through Resonance
M. D. Todd, L. N. Virgin, and J. A. Gottwald, Duke University, Durham, NC

Nonstationary Response of Rössler's Folded Band
K. Rangavajhua and H. G. Davies, University of New Brunswick, Fredericton, New Brunswick, CANADA

Parametric Random Excitation of Nonlinear Coupled Oscillators
Y. J. Yoon and R. A. Ibrahim, Wayne State University, Detroit, MI

Finite Element Nonlinear Random Response Analysis of Composite Plates to Normal/Grazing Incidence Wave
R. R. Chen and C. Mei, Old Dominion University, Norfolk, VA

1510-1530 Break

Session 2. Control

Chairmen: H. G. Davies, University of New Brunswick, Fredericton, New Brunswick, CANADA and F. Golnaraghi, University of Waterloo, Waterloo, Ontario, CANADA

1530-1710

Active Vibration Suppression Using a Rotational Translation Actuator
V. T. Coppola, University of Michigan, Ann Arbor, MI

Adaptive Dynamics of a Spring Supported Truss Member
H. W. Haslach, Jr., University of Maryland, College Park, MD

The Effects of Nonlinearities Upon Fuzzy Structural Control
F. Casciati, L. Faravelli, and T. H-J. Yao, University of Pavia, Pavia, ITALY

Robust Control for a Hybrid Nonlinear Partial Differential Equation
J. A. Burns, Virginia Polytechnic Institute and State University, Blacksburg, VA and B. B. King, Oregon State University, Corvallis, OR

Direct Optimal Nonlinear Control of the Duffing Dynamics
H. Öz and E. Adigüzel, The Ohio State University, Columbus, OH

1900-2100 Reception

Session 3. Modal Interactions

Chairmen: A. K. Bajaj, Purdue University, West Lafayette, IN and M. R. M. Crespo da Silva, Rensselaer Polytechnic Institute, Troy, NY

0830-1010

Bispectral Analysis of Nonlinear Interactions
B. Balachandran and P. F. Cunniff, University of Maryland, College Park, MD

Experimental Investigation of Random Excitation of Coupled Beams System with Multiple Internal Resonances
A. A. Afaneh and R. A. Ibrahim, Wayne State University, Detroit, MI

Nonstationary Oscillations of an Orbiting String in Conditions of Multiple Internal Resonances
A. Luongo, F. Vestroni, and A. Di Egidio, Universitá L'Aquila, L'Aquila, ITALY

On the Hardware Development of an Internal Resonance Controller
S. Oueini and M. F. Golnaraghi, University of Waterloo, Waterloo, Ontario, CANADA

Multimodal Resonances of Elastic Structures
K. Yasuda, Nagoya University, Nagoya, JAPAN

1010-1030 Break

Session 4. Parametric Vibrations

Chairmen: R. A. Ibrahim, Wayne State University, Detroit, MI and G. T. Flowers, Auburn University, Auburn, AL

1030-1210

The Influence of Parametric Vibrations on Nonlinear Oscillators and their Control
F. Colonius, Universität Augsburg, Augsburg, GERMANY and W. Kliemann, Iowa State University, Ames, IA

On a Parametrically Excited Extensible Pendulum
D. J. Shippy and H. Fu, University of Kentucky, Lexington, KY

Analysis of Nonlinear Time-Periodic Dynamical Systems Under Critical Conditions
R. Pandiyan and S. C. Sinha, Auburn University, Auburn, AL

Nonlinear Analysis of Subharmonic Parametric Resonances of a Cantilevered Pipe Conveying Fluid
M. P. Paidoussis and C. Semler, McGill University, Montreal, Quebec, CANADA

Theoretical and Experimental Investigation of the Response of Initially Curved Rectangular Plates
S. Sassi and G. L. Ostiguy, Ecole Polytechnique, Montreal, Quebec, CANADA

1210-1330 Lunch

Session 5. Impact and Friction

Chairmen: S. W. Shaw, Michigan State University, East Lansing, MI and D. Beale, Auburn University, Auburn, AL

1330-1510

Dynamic Analysis of Rotary-Augmenter Device and Optimum Design of Rubber Damper of Blast Hole Drills Machine
R. Liyi, Northeastern University, Liaoning, PRC and J. Youxin, Shenyang Industry University, Shenyang, PRC

Multiple Impacts with Friction in Rigid Multibody Systems
Ch. Glocker and F. Pfeiffer, Technische Universität München, München, GERMANY

Nonlinear Elastic Dynamic Contact Problems in Travelling Wave Ultrasonic Motors
J. Wallaschek, University of Paderborn, Paderborn, GERMANY

Dynamics of Flexible Mechanical Systems with Contact-Impact and Plastic Deformations
J. P. Dias and M. S. Pereira, Instituto Superior Técnico, Lisboa, PORTUGAL

Motions of a Mass-Spring System Between Two Rigid Asymmetric Barriers, Stability and Unstability
J. Anglés, EDF-DER, Clamart, FRANCE

1510-1530 **Break**

Session 6. Rotor and Structural Dynamics

Chairmen: F. Vestroni, Universitá L'Aquila, L'Aquila, ITALY and D. H. van Campen, Eindoven University of Technology, Eindoven, THE NETHERLANDS

1530-1710

On the Counteraction of Periodic Torques in Rotating Systems by Means of Centrifugally Driven Vibration Absorbers
S. W. Shaw, Michigan State University, East Lansing, MI and C-T. Lee, University of Michigan, Ann Arbor, MI

Forced Oscillations of a Vertical Continuous Rotor with Geometrical Nonlinearity
Y. Ishida, I. Nagasaka, T. Inoue, and S. Lee, Nagoya University, Nagoya, JAPAN

Nonlinear Response of Rotors Supported on Journal Bearings
P. Sundararajan and S. T. Noah, Texas A&M University, College Station, TX

Nonlinear Dynamics of Rotor Systems Experiencing Rubbing
F. Wu and G. T. Flowers, Auburn University, Auburn, AL

Invariant Manifolds, Nonlinear Vibrations in a Singularly Perturbed Nonlinear Oscillator with Applications to Structural Dynamics
I. T. Georgiou, M. J. Corless, and A. K. Bajaj, Purdue University, West Lafayette, IN

Session 7. Computational Methods

Chairmen: K. Yasuda, Nagoya University, Nagoya, JAPAN and W. Kliemann, Iowa State University, Ames, IA

0830-1010

A Comparison of the Global Convergence Characteristics of Some Fixed Point Methods
M. D. Conner, P. Donescu, and L. N. Virgin, Duke University, Durham, NC

Efficient Simulation of Systems with Discontinuities and Time-Varying Topology
J. P. Meijaard, Delft University of Technology, Delft, THE NETHERLANDS

Nonlinear Structural Response Using Adaptive Dynamic Relaxation on a Massively-Parallel-Processing System
D. R. Oakley, Clemson University, Clemson, SC and N. F. Knight, Jr., Old Dominion University, Norfolk, VA

An Iterative Scheme of Point Mapping Under Cell Reference for Global Analysis
J. Jiang and J. Xu, Xi'an Jiaotong University, Xi'an, PRC

Panel Flutter Studies in Hypersonic Flow Based on the Navier Stokes Equations
S. R. Sipcic and L. Morino, Boston University, Boston, MA

1010-1030 **Break**

Session 8. Bifurcations

Chairmen: A. C. Soudack, The University of British Columbia, British Columbia, CANADA and M. P. Paidoussis, McGill University, Quebec, CANADA

1030-1210

Bifurcations in Planar Piecewise Linear Systems
S-L. Chen, Michigan State University, East Lansing, MI

Jumps to Resonance with a Probabilistic Outcome in Systems Subjected to Deterministic Excitation
M. S. Soliman, University of London, London, UNITED KINGDOM

Routes to Escape from a Potential Energy Well Including Experiments
J. A. Gottwald, L. N. Virgin, and E. H. Dowell, Duke University, Durham, NC

Modeling and Bifurcations in Power System Dynamics
A. M. A. Hamdan, Jordan University of Science and Technology, Irbid, JORDAN

Bifurcation and Chaos in the Duffing Oscillator with a PID Controller
F. Cui, J. Xu, and Y. Cai, Xi'an Jiaotong University, Xi'an, PRC

1210-1330 Lunch

Session 9. Chaos

Chairmen: F. Casciati, University of Pavia, Pavia, ITALY and A. M. A. Hamdan, Jordan University of Science and Technology, Irbid, JORDAN

1330-1510

On the Period-Doubling Bifurcations in the Duffing's Oscillator with Negative Linear Stiffness
K. R. Asfar and K. K. Masoud, Jordan University of Science and Technology, Irbid, JORDAN

Chaotic Unpredictability of Elastic-Plastic Response to Impact Loading
P. S. Symonds and Y. Qian, Brown University, Providence, RI

Global Bifurcations and Chaos in the Resonant Response of a Structure with Cyclic Symmetry
S. Samaranayake, A. K. Bajaj, and O. D. I. Nwokah, Purdue University, West Lafayette, IN

Nonlinear and Chaotic Dynamics of Articulated Cylinders in Confined Axial Flow
R. M. Botez and M. P. Paidoussis, McGill University, Montreal, Quebec, CANADA

Chaotic Dynamics of Quadratic Systems with 1:2 Internal Resonances
B. Banerjee and A. K. Bajaj, Purdue University, West Lafayette, IN

1510-1530 **Break**

Session 10. Multibody Dynamics

Chairwoman and Chairman: L. Faravelli, University of Pavia, Pavia, ITALY and H. M. Lankarani, Wichita State University, Wichita, KS

1530-1710

Experimental Study of a Complex Nonlinear Mechanical System
M. Boltezar, University of Ljubljana, Ljubljana, SLOVENIA and J. K. Hammond, University of Southampton, UNITED KINGDOM

A Nonlinear Finite Approach for Kineto-Static Analysis of Multibody Systems
D. Ma and H. M. Lankarani, Wichita State University, Wichita, KS

A Symbolic-Numerical Approach to Characterize the Stability and Control the Dynamics of a Four-Wheel-Steering Vehicle
N. E. Sanchez, University of Texas at San Antonio, San Antonio, TX

Symbolic Modelling of Flexible Robots and Identification of Dynamic Parameters
P. Depince and P. Chedmail, Ecole Centrale de Nantes et Université de Nantes, Nantes, FRANCE

Experimental and Numerical Investigation of the Pitch and Bounce Response of a Railroad Vehicle
W. P. O'Donnell, Association of American Railroads, Chicago, IL and A. A. Shabana, University of Illinois, Chicago, IL

1900 **Banquet**

Session 11. Analytical Methods I

Chairmen: S. C. Sinha, Auburn University, Auburn, AL and D. Gilsinn, National Institute of Standards and Technology, Gaithersburg, MD

0830-1010

Control of Dynamical Systems Subjected to Periodic Parametric Excitations
R. Pandiyan, P. Joseph, and S. C. Sinha, Auburn University, Auburn, AL

Spurious Solutions Predicted by the Harmonic Balance Method
A. Hassan and T. D. Burton, Washington State University, Pullman, WA

A Renovated Algorithm for Incremental Harmonic Balance Method
T. Ge and A. Y. T. Leung, University of Hong Kong, HONG KONG

On the Accuracy of the "Selected Block" Approach to the Local Stability Analysis of the Approximate Harmonic Balance Solutions
A. Hassan, Washington State University, Pullman, WA

Lie-Transformation Method for Dynamics and Control of Weakly Nonlinear Autonomous Systems
L. Morino, Terza Universitá di Roma, Rome, ITALY and F. Mastroddi, Universitá di Roma "La Sapienza", Rome, ITALY

1010-1030 **Break**

Session 12. Analytical Methods II

Chairmen: J. Wu, Army Research Office, Research Triangle Park, NC and D. J. Shippy, University of Kentucky, Lexington, KY

1030-1210

Constructing Galerkin's Approximations of Invariant Tori Using MACSYMA
D. Gilsinn, National Institute of Standards and Technology, Gaithersburg, MD

The Dynamics of Resonant Capture
D. Quinn, R. Rand, Cornell University, Ithaca, NY, and J. Bridge, Georgia Institute of Technology, Atlanta, GA

Nonlinear Parametric Identification by Balancing Harmonics of Extracted Periodic Orbits
B. F. Feeny and C. -M. Yuan, Michigan State University, East Lansing, MI

Using Adjoint Operator Method to Compute Normal Form of Order 4 for Nonlinear Dynamical System
W. Zhang, Tianjin Institute of Technology, Tianjin, PRC

Improving the Equivalent Linearization for Stochastic Duffing Oscillator
J. Lee, Wright-Patterson Air Force Base, OH

1210-1330 **Lunch**

Session 13. Structural Dynamics I

Chairmen: P. Meijers, Delft University of Technology, Delft, THE NETHERLANDS and S. Noah, Texas A&M University, College Station, TX

1330-1510

Appropriate Stress and Strain Measures for Nonlinear Structural Analyses
P. F. Pai, North Carolina A&T State University, Greensboro, NC

Secondary System Analysis for Space Pay-Load
M. Battaini, F. Casciati, and L. Faravelli, University of Pavia, Pavia, ITALY

Dynamic and Thermal Response of Space Payload Structures
I. I. Orabi, University of New Haven, West Haven, CT and R. B. Malla, University of Connecticut, Storrs, CT

Nonlinear Finite Element Behavior of Cooling Towers
S. J. Serhan, Gilbert/Commonwealth, Inc., Reading, PA

3D Finite Element Modeling and Analysis of Armored Vehicle Hulls with Multiple Access Openings
A. D. Gupta, J. M. Santiago, and C. Meyer, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD

1510-1530 **Break**

Session 14. Localization and Normal Modes

Chairmen: D. Inman, Virginia Polytechnic Institute and State University, Blacksburg, VA and R. H. Rand, Cornell University, Ithaca, NY

1530-1710

Modal Analysis for Non-Linear Structural Systems
N. Boivin, C. Pierre, and S. W. Shaw, The University of Michigan, Ann Arbor, MI

Localized and Non-Localized Nonlinear Normal Modes in a Multi-Span Beam with Geometric Nonlinearities
J. Aubrecht and A. F. Vakakis, University of Illinois at Urbana-Champaign, Urbana, IL

A Numerical Method for Determining Nonlinear Normal Modes
J. C. Slater, Wright State University, Dayton, OH

On Nonlinear Normal Modes of Systems with Internal Resonance
A. H. Nayfeh, C. Chin, Virginia Polytechnic Institute and State University, Blacksburg, VA, and S. A. Nayfeh, Massachusetts Institute of Technology, Cambridge, MA

Dynamics of a Mono-Coupled Elastic Periodic System with Material Nonlinearities
A. F. Vakakis and M. E. King, University of Illinois at Urbana-Champaign, Urbana, IL

Session 15. Structural Dynamics II

Chairmen: Y. Ishida, Nagoya University, Nagoya, JAPAN and K. R. Asfar, Jordan University of Science and Technology, Irbid, JORDAN

0830-1010

Statistical Properties of Nonlinear Vibrations of Elastic Beams
V. L. Berdichevsky, E. Mueller, A. Özbok, and I. Shektman, Georgia Institute of Technology, Atlanta, GA

Thermodynamics of Chaotic Structural Dynamic Systems
S. Hanagud and L. N. B. Gummadi, Georgia Institute of Technology, Atlanta, GA

Dynamics of an Elastic Rod in a Fluid Pumping System
D. Beale, Auburn University, Auburn, AL

On the Quasi-Steady Analysis of One-Degree-of-Freedom Galloping with Combined Translational and Rotational Effects
B. W. van Oudheusden, Delft University of Technology, Delft, THE NETHERLANDS

An Experimental Investigation of Energy Transfer from a High-Frequency Mode to a Low-Frequency Mode in a Flexible Structure
P. Popovic, A. H. Nayfeh, K. Oh, Virginia Polytechnic Institute and State University, Blacksburg, VA, and S. A. Nayfeh, Massachusetts Institute of Technology, Cambridge, MA

1010-1030 **Break**

Session 16. Structural Dynamics III

Chairmen: R. M. Evan-Iwanowski, University of Central Florida, Orlando, FL and L. N. Virgin, Duke University, Durham, NC

1030-1210

Experimental and Analytical Investigations of the Nonlinear Response of a Cantilever Under Transverse Excitation
M. R. M. Crespo da Silva, Rensselaer Polytechnic Institute, Troy, NY

Dynamics of a Multi-DOF Beam System with Discontinuous Support
D. H. van Campen, E. L. B. van der Vorst, A. de Kraker, and R. H. B. Fey, Eindhoven University of Technology, Eindhoven, THE NETHERLANDS

Nonlinear Vibrations in Beams and Frames: The Effect of the Deformed Equilibrium State
J. C. André, Escola Politécnia da Universidade de São Paulo, São Paulo, BRAZIL

Nonlinear Dynamics of a Cantilever Beam Carrying a Moving Mass
S. Rajagopalan, M. F. Golnaraghi, and G. R. Heppler, University of Waterloo, Waterloo, Ontario, CANADA

Vibrations of a Portal Frame Excited by a Non-Ideal Motor
R. M. L. R. F. Brasil and D. T. Mook, Virginia Polytechnic Institute and State University, Blacksburg, VA

Sunday, June 12

1320-1330 Opening Remarks

1330-1510

Session 1. Nonstationary and Random Vibrations

Nonstationary (NS) Food for Thought

R. M. Evan-Iwanowski, University of Central Florida
Chu Ho Lu, Memphis State University
G. L. Ostiguy, Montreal Polytechnical Institute

A varied menu of nonstationary (NS) dynamical and chaotic behavior is presented in this paper: it is revealing, puzzling but always interesting. In NS systems some parameters (P) of the governing operators depend on the physical processes τ (time, heat, temperature, electrical charge, viscosity, etc.): $P_{NS} = P_0 + \psi(\tau)$, and they also satisfy codimension relationships $\phi(P_1, P_2, \ldots, P_n) = 0$. The functions $\psi(\tau)$, called the process functions indicate the effects of the processes τ on the control parameters P, and the functions ϕ indicate the paths in the P-space. The functions ψ and ϕ are arbitrary: continuous, discrete, random, imperical, etc. To demonstrate the role of the functions ψ and ϕ, consider stationary (ST) $2T$-line (codimension one) found by Sanchez and Nayfeh in the (forcing frequency ν; forcing amplitude f) $= (\nu, f)$-space. We extended the $2T$ line to the ST period doubling bifurcation region converging to ST chaos. First we used the line $2T$ or L-line (the function ϕ), and then the line perpendicular to it or E-line (function ϕ_2). We set $f_{NS} = f_0 \pm \alpha t$, $\nu_{NS} = \nu_0 \pm \alpha t$, $\alpha =$ constant, along these lines. New and different NS dynamical and chaotic responses have been obtained, Figs. 1, 2: In both cases, the NS responses *converged to the NS limit motion*, appearing to be NS chaos. The NS chaos along the E-line preceded both the ST and the NS chaos along the L-line.

The NS effects on the ST bifurcations can be adequately illustrated using normal, one-parameter ST bifurcations. In Fig. 3 are presented the results for the following $\psi(\tau)$ process functions: linear $\psi_l(\tau) = \pm\alpha t$, *cyclic* $\psi_C(\tau) = \gamma \sin \beta t$; exponential $\psi_e(\tau) = \exp(-mt)$; and impulse $\psi_i(\tau) = B\,\delta(t - t_1)$ where $\alpha, \gamma, \beta, A, m$ and B are constant and "δ" is the Kronecker delta. *The NS inputs strongly affect the ST bifurcations*, particularly the NS cyclic input shows its dominance: it enforces the cyclic response right at the moment of its application regardless of the ST system.

Clearly, the NS systems exhibit *transient behavior* because of the presence of the functions ψ and ϕ, and they are clearly *complex* for the same reason. Guided by a thought that in every complexity a settled down pattern may be found, the authors initatiated a diligent search for such unique episodes. In the Duffing (Hayashi form) oscillator $\ddot{x} + c\dot{x} + dx^3 = f \cos\theta(t)$, we let $\dot{\theta}(t) = \nu(t) = \nu_0 + \alpha t$, where ν_0 is the initial perturbation on the ST chaotic amplitude-time plot. Fig. 4 presents the NS panoramic view of the transition from NS chaos to a series of NS settled down, periodic behavior. It looks as though the NS responses were "*slowed down*" near the NS bifurcations. Of interest is the appearance of the continually changing responses referred to as *transient or flight curves* Fig. 4. A few words on the transiency are in order. Most commonly the notion of transient phenomena are related to the existence of viscouse or generally dissipative mechanisms, whereby the responses tend to zero. Also, some initial computational responses are being discarded as "transient." Often, a transitional state between two different states, e.g. solid and molten, is referred to as a mash. In the NS space, the transiency acquire a definitive meaning of NS "flight curves" between the bifurcations or attractors.

A startling observation has been made in connection with the following: some responses obtained for *different* ODEs, different PS and different α's appear to be identical, Fig. 5.

A far reaching observation has been made regarding the appearance of the *three distinct categories* (L), L1, L2 and L3 of the linear NS responses, $\psi_l(\tau) = \pm\alpha t$, and two distinct categories (C), C1 and C2 of the NS cyclic responses, $\psi_c(\tau) = \gamma \sin \beta t$, Fig. 6. What is puzzling that the same categories L or C appear in the different systems - beams, columns, plates, shells, rotors - metallic or composite, and for different types of resonances - dynamic (forcing), parametric, combination additive (sum) and differential (difference). These categories consistently correlate with the

constants appearing in the ψ and ϕ functions and the initial conditions. In some exceptional cases, however, the *correlation* may be *violated.* For example, the category L1 is usually associated with the low values of α, and the category L3 with the high α. Fig. 6b shows the contradictory results: the category L1 appears for higher values of α than for the L3 category. The loci separating the categories are difficult to calculate. They are, obviously, very important for the analyst-designer to determine beforehand specific modes of operation.

It has been shown by Mosheby and Evan-Iwanowski that the NS responses for $\psi(\tau) = \pm\alpha t$ which started at a point of the ST chaos plot, initially coincide with the ST chaos, then they depart from it: the larger the values α the sooner is the departure. This observation suggests that the ST Lyapunov exponents λ need to be replaced by *NS values* $\lambda(t)$.

One of the fundamental questions is: does the NS responses *converge* to the corresponding ST response; i.e., $\lim\limits_{\psi(T)\to 0} NS = \underset{\psi(T)=0\}}{ST}$. The results, so far, are contradictory (Virgin & Thompson vs. A. D. Johnson).

The results obtained by Szemplinska-Stupnicka presented for discussion at the "Time-Varying Symposium, ASME 1993" related to the appearance of *the NS components in ST chaos,* enhances further the applicability of the NS formulations.

A video presented by Cusumano at the 1990 VPI Conference on the depletion of the domain of attraction by the quasi-NS input, opens a fertile ground for further interpretations of the role of the functions ψ and ϕ in NS dynamics and chaos.

A problem of small divisor which plagued the world savants before the proof of the KAM theorem, did not and does not <u>exist</u> in the NS environment. Is there food for thought? Like this one: the universality of the nonstationarity (the functions ψ and ϕ) in the physical world. Qui vivra, vera. Fig. 7 shows NS window in the chaotic (weather) range of the Lorenz System.

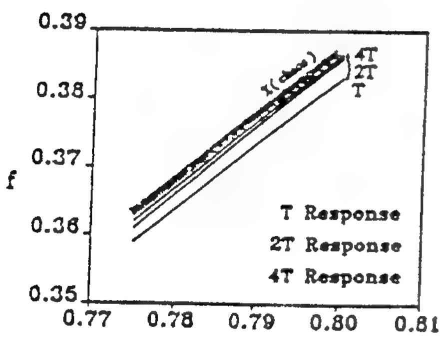
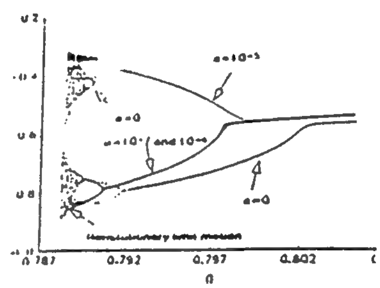
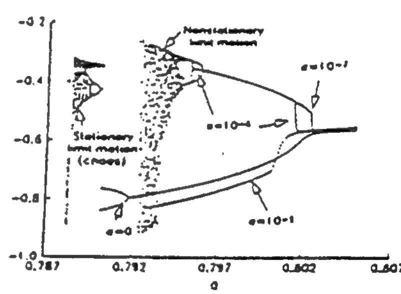


Fig. (1-2)



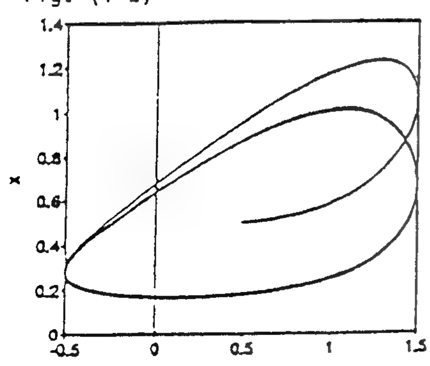
ST bifurcations for $f = -4.016 + \Omega$

Fig. 1



NS bifurcations for $f(t) = -4.016 + \Omega(t)$, $\dot\theta(t) = \Omega(t)$

Fig. 2



$u = 0.5 + \sin\tau, x(0) = 0.5$

Fig. 3

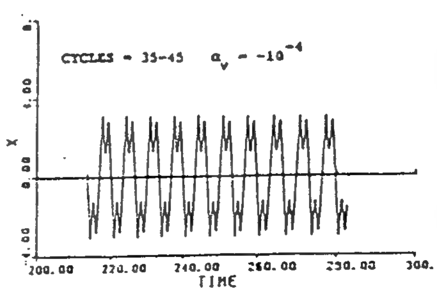


Fig. 4a

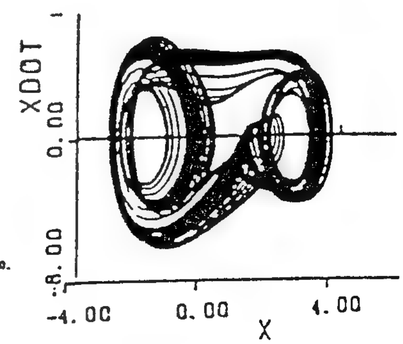


Fig. 4b

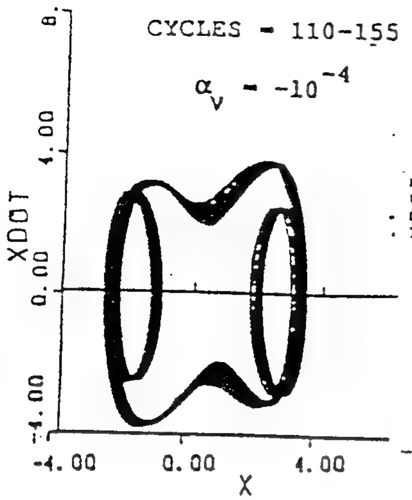


Fig. 4c

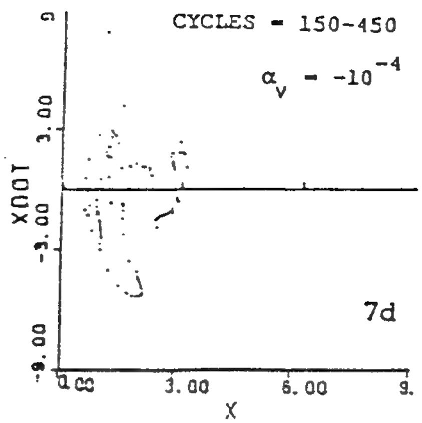


Fig. 4d

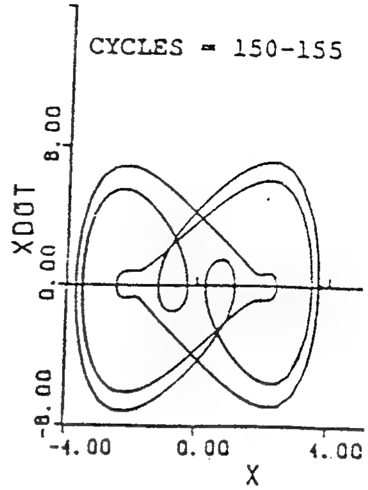


Fig. 4e

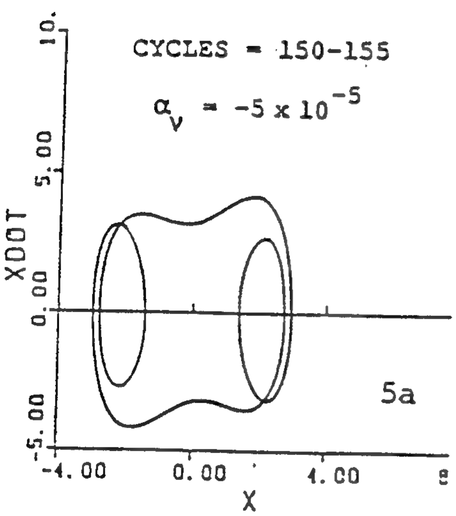


Fig. 5a

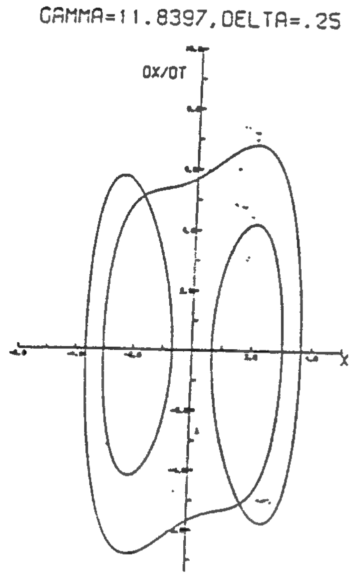


Fig. 5b

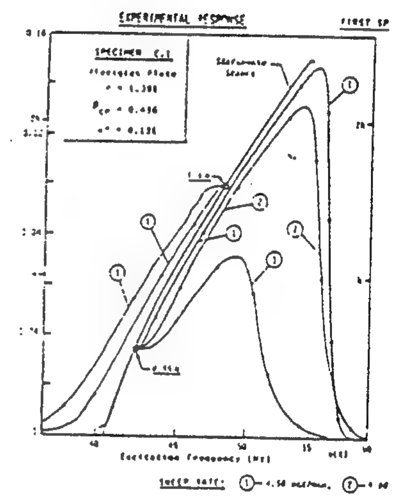


Fig. 6a

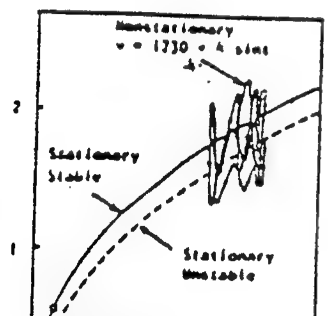


Fig. 6b

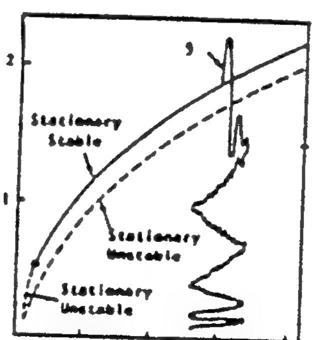


Fig. 6c

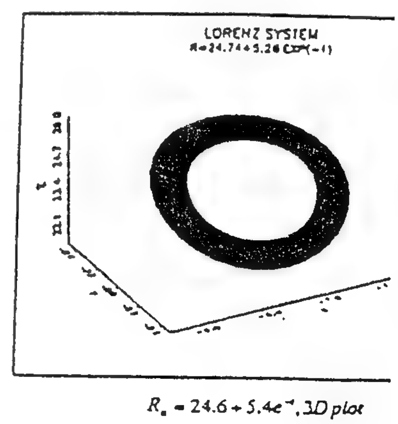


Fig. 7

Nonstationary transition through resonance

by

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Abstract

This paper considers the resonant behavior of a mechanical oscillator during a slow frequency sweep. Both numerical and experimental results are presented. The experimental system, consisting of a track in the shape of a potential energy surface, has been used to highlight other types of nonlinear behavior and is here adapted so that the forcing frequency can be evolved continuously. The classic linear oscillator (with a parabolic potential well) is used as an introduction to illustrate basic features of the experiment and its response. Then, a track with a double well is used to assess nonstationary frequency effects on certain nonlinear resonance characteristics.

INTRODUCTION

There are many examples of nonstationary dynamic systems in applied mechanics where certain parameters are time-varying. The classic examples are a rocket expending fuel and hence losing mass, and the passage of a rotating shaft through a critical speed. There are a variety of other problems where parameters gradually change. The extent to which this nonstationarity is important clearly depends on the rate of changing conditions relative to certain fundamental characteristics of the system. The work described in this paper considers an intermediate case where the frequency is a linear function of time. This gradual evolution is slow, but non-negligible over the time scales considered. In this case recourse is made to numerical techniques, since

although the governing equations may be well-defined, analytical solutions are limited, especially for nonlinear systems.

Previous research in this area began with the study of critical speeds in rotating systems (Lewis, 1932) and more recent work on linear dynamical systems includes transient testing using frequency sweeping (White, 1971), and resonant turbine blade behavior (Irretier and Leul, 1991). The effect of non-stationary influences on nonlinear systems includes predicting instabilities using transient dynamic effects (Virgin, 1986), nonlinear resonant effects in rotating shafts (Ishida *et al.*, 1987), approximate analytical results based on the perturbation method (Raman *et al.*, 1993), and chaotic behavior (Moslehy and Evan-Ivanowski, 1991).

The experimental system used to illustrate this type of behavior has been used successfully to illustrate a variety of nonlinear behavior (Gottwald *et al.*, 1992). Specifically the influence of frequency sweep rate on the resonant characteristics (flip bifurcations and jumps) of the peak amplitude response is investigated and a comparison is made between experimental results and numerical simulation.

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Nonstationary Response of Rössler's Folded Band

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Rössler's or the folded band attractor models the folding of trajectories in phase space that is exhibited by the Lorenz attractor, but in a simpler form. It is probably the simplest three dimensional vector field that generates a folding effect that can lead to chaos.

We consider the system in the form

$$\dot{x} = -y - z$$

$$\dot{y} = x + 0.2y$$

$$\dot{z} = 0.2 + z(x - \mu)$$

μ is a control parameter. As the value of μ is increased, trajectories in (x,y,z) phase space demonstrate period doubling and chaotic response, followed as μ is increased further by a variety of periodic and chaotic responses. The largest Lyapunov exponent first becomes positive at a value $\mu \approx 4.2$. Typical trajectories are shown in the text by Thompson and Stewart [1]. Calculations of the power spectral density of the response by Crutchfield and others [2] show very strong periodic components in the response, even in chaotic regions.

In this paper we consider the nonstationary response of this system caused by varying μ either linearly or periodically. The rate of variation of μ is kept small, much smaller than the typical fundamental period of the trajectories, although the rate may not be small relative to the relaxation time or rate at which solutions are attracted to a stable periodic orbit. A Poincaré map is obtained by sampling y and z for $x = 0$, $y > 0$. On this plane the values of z are all small. The response can be characterised as in Figures 1-4 by plotting y as a function of μ.

Figures 1 and 2 show linear variation of μ in the form $\mu = \mu_o + \mu_1 t$. For $\mu_1 > 0$, the bifurcations from period -1 to 2 and from 2 to 4 are delayed, although the first onset of chaos occurs close to the stationary value. (The dashed lines show the stationary period -1, 2 and 4 response.) For $\mu_1 < 0$, the region of chaotic response extends below the stationary value of $\mu = 4.2$. However, periodic response such as periods -4 and 6 is more pronounced, again indicating the superposition of periodic response within the chaotic regions.

Figures 3 and 4 show periodic variation of μ in the form $\mu = \mu_o + \mu_1 \cos \omega t$. The form (period -1 or -2) of the response depends on μ_o, and the bifurcation takes place at the static value. These periodic solutions show remarkable, apparently global, stability, although they may be locally unstable in some regions.

The thrust of this work is to analyse the stability of these nonstationary responses, and, by appropriate reductions to normal forms, to analyse the possible shifts in bifurcation values.

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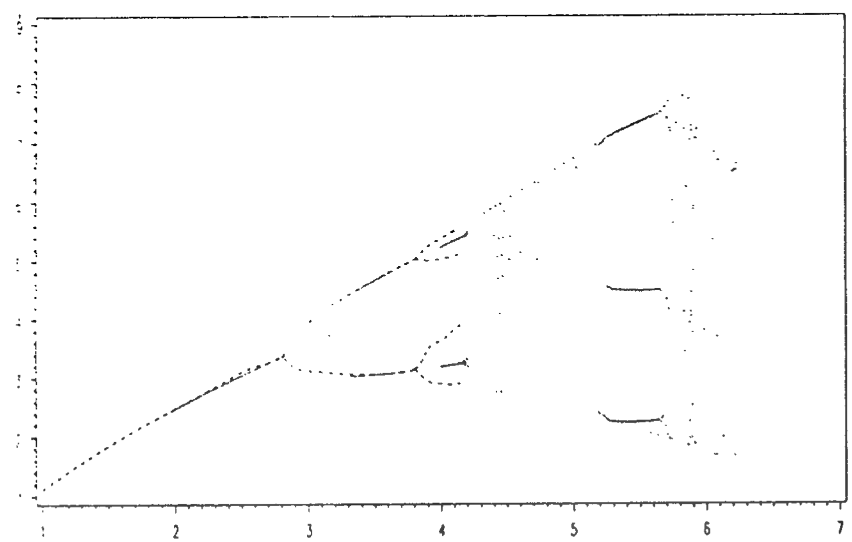

Fig. 1: $\mu = \mu_o + 0.001 t$

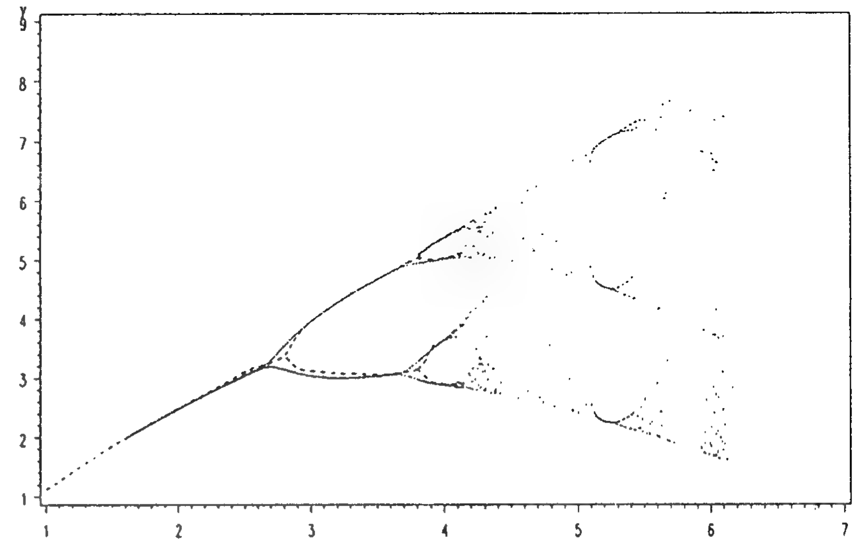

Fig. 2: $\mu = \mu_o - 0.001 t$

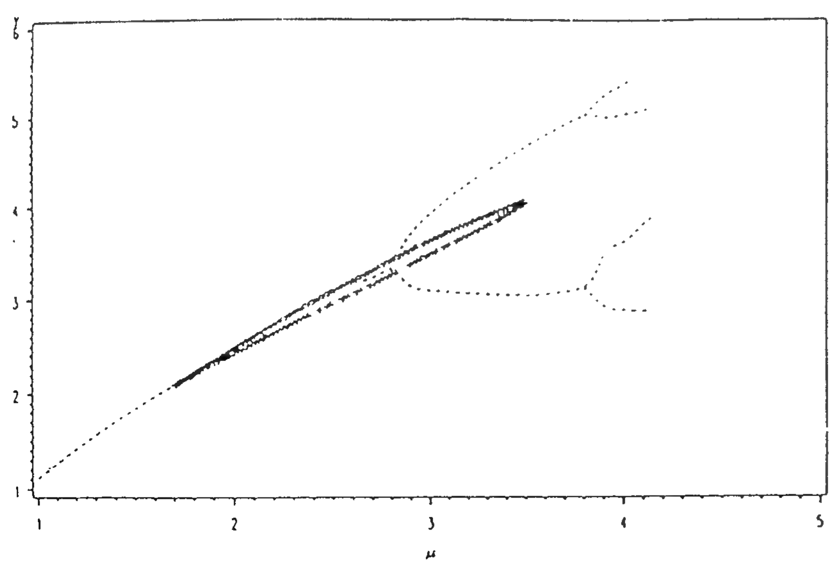

Fig. 3: $\mu = 2.6 + 0.9 \cos (0.05t)$

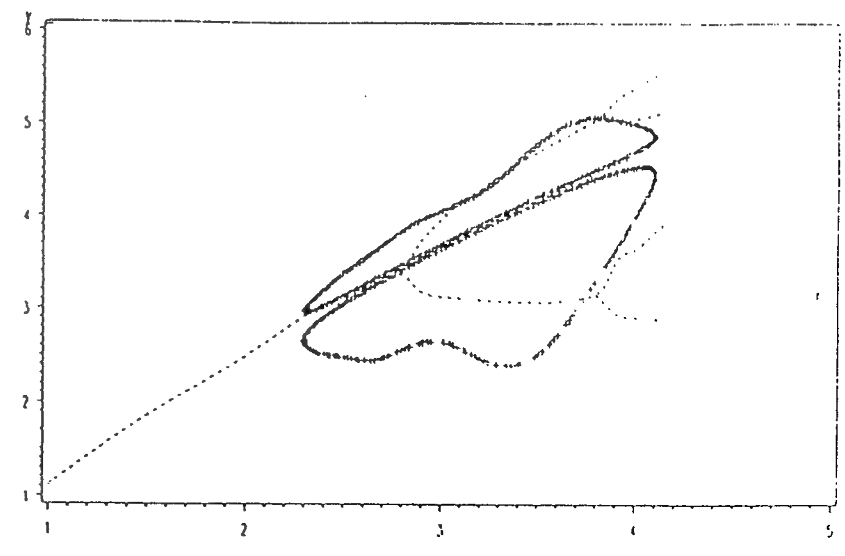

Fig. 4: $\mu = 3.2 + 0.9 \cos (0.05t)$

PARAMETRIC RANDOM EXCITATION OF NONLINEAR COUPLED OSCILLATORS

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ABSTRACT

The stochastic bifurcation and response statistics of nonlinear modal interaction under parametric random excitation are studied analytically, numerically, and experimentally. Two basic definitions of stochastic bifurcation are first introduced. These are bifurcation in distribution and bifurcation in moments. Bifurcation in moments is examined for the case of a coupled oscillator subjected to parametric filtered white noise. The center frequency of the excitation is selected to be close to either twice the first mode or second mode natural frequencies or the sum of the two. The stochastic bifurcation in moments is predicted using the Fokker-Planck equation together with Gaussian and non-Gaussian closures and numerically using the Monte Carlo simulation. When one mode is parametrically excited it transfers energy to the other mode due to nonlinear modal interaction. The Gaussian closure solution gives close results to those predicted numerically only in regions well remote from bifurcation points. However, bifurcation points predicted by the non-Gaussian closure are in good agreement with those estimated by numerical simulation. Depending on the excitation level, the probability density of the excited mode is strongly non-Gaussian and exhibits multi-maxima as predicted by Monte Carlo simulation. Experimental tests are carried out at relatively low excitation levels. In the neighborhood of stochastic bifurcation in mean square the measured results exhibit different regions of response characteristics including zero motion and occasional small random motion regimes. Both regimes overlap and thus it is difficult to locate experimentally the bifurcation point.

Finite Element Nonlinear Random Response Analysis of Composite Plates to Normal/Grazing Incidence Wave

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ABSTRACT

For future high speed civil transportation vehicles, the jet exhaust velocities at take-off will be about 400 m/s (1312 fps), it is greater than the sound speed. Obviously the structures near by will be subjected to severe acoustic loads. Under these severe loads, the structures will go to large-amplitude random vibrations. On the other hand, in order to meet increased performance requirements, new complex, lightweight structures and advanced materials will be required. The complex structures under consideration have significant uncertainties in fatigue behavior due to intense acoustic loads. The intense acoustic loads can affect fatigue life by introducing large deflection geometrical nonlinearities, modal coupling and multiple-mode participation. Such high sound pressure levels may even drive the structures to have damping nonlinearity. Because of high costs and difficulties with instrumentation in experiments at high acoustic intensity, reliable experimental data is difficult to acquire. Thus, in the design process, greater emphasis will be placed on analytical and computational methods. This brings a tremendous challenge to the analysts for predicting nonlinear response of complex structures subjected to high level acoustic loads.

From the literature survey, it appears that studies of the nonlinear random response of plates subjected to a grazing incidence acoustic wave using the finite element method are not available. Therefore, this paper presents a finite element formulation and solution procedure which combine the equivalent linearization technique and the normal mode method for the analysis of nonlinear random response of composite panels to normal/grazing incidence acoustic wave. The grazing wave model used in this study is suitable to simulate the acoustic waves in the progressive wave test facility.

The formulation is based on the von Karman large deflection plate theory and the first-order transverse shear deformation theory. The element stiffness matrix used for this study is developed and provided by Tessler and Hughes. It is a three-node Mindlin (MIN3) plate element with improved transverse shear. In this paper, it is extended and is used for this study. The extension includes the development of the mass matrix, the nonlinear stiffness matrices and the load vectors. It is assumed that the grazing wave

moves in the direction θ with speed c, but the pressure at any point is random. The pressure distribution on a plate is then given by

$$p(x,y,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\omega) e^{i\omega\left(t - \frac{x}{c}\sin\theta\right)} d\omega \tag{1}$$

where x is the coordinate along the wave travelling direction, and assume that the pressure distribution is independent of y.

The results are very interesting. Because the acoustic wave is travelling along the positive direction of x axis with a speed c, the acoustic pressure on the plate is no longer uniform, the anti-symmetric modes participates in the response of the plate. Therefore the maximum deflection point moves forward slightly as shown in Fig. 1. Figure 2 shows the maximum deflection spectrum to normal/grazing incidence acoustic loads. Both responses use two modes, for grazing incidence utilizes the 1st and 2nd modes; and for normal incidence uses the 1st and 3rd modes. Interestingly, the first peak frequency shifted up quite differently. It is due to different strain distribution, therefore different stiffening effect.

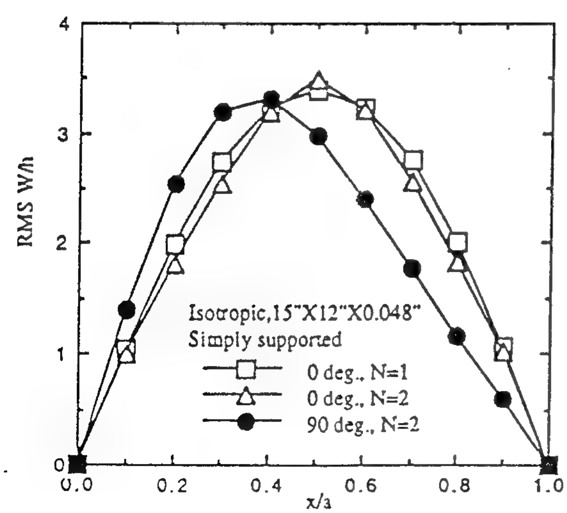

Fig. 1 Distributions of RMS W/h along the center line

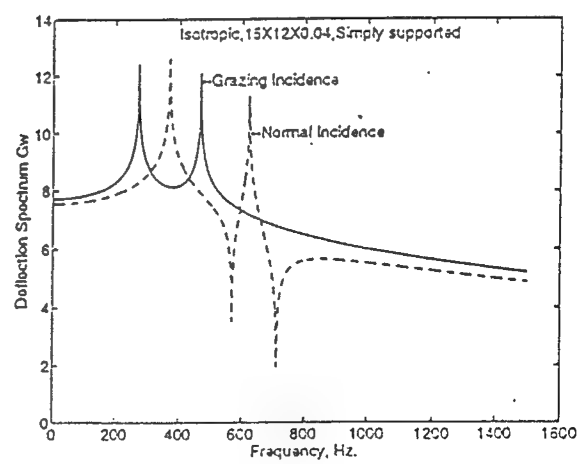

Fig. 2 The maximum deflection spectrum to normal incidence and grazing incidence loads at 130dB

Sunday, June 12

1530-1710

Session 2. Control

Active Vibration Suppression using a Rotational Translation Actuator

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Dynamic vibration absorbers are devices which reduce undesirable vibrations in machinery experiencing oscillatory forcing. The simplest vibration absorber for machines modeled as linear mass-spring systems is another mass-spring system connected in series, referred to as a Den Hartog absorber. When properly tuned, the Den Hartog absorber reduces the vibration amplitude of the main system for forcing excitations near the machine's resonance. Yet, for other forcing frequencies the absorber system results in enhancement of of the machine vibration amplitude. Thus, this simple absorber is beneficial only for narrow-band excitations.

Many other absorber strategies have been proposed over the years to improve upon the Den Hartog absorber. The designs have been focussed on using a passive device as an absorber. This avoids the complexity that an active device would require, i.e., a sensor, an actuator, and an external power supply. However, current technology suggests that the simplicity advantage of a passive device may be over-rated. In fact, active control devices have been explored for use in suppressing vibrations of flexible spacecraft because the performance of an active device can outweigh the costs of complexity.

This has led to the development of the proof-mass actuator, an active control device which suppresses translational vibration through the linear motion of a small proof-mass. The device requires a linear actuator which tends to be expensive and difficult to build. In addition, the device is stroke-limited (i.e., the mass must translate within the confines of the device) to avoid impact with the housing.

In contrast, rotational actuators are readily available in the form of the common motor. This led us to ask:

Can rotational motion generated by a motor be used to suppress translational vibrations?

We have found the answer to be *yes*, provided that a proper feedback controller be used to proscribe the motor torque.

Thus, we present a preliminary analysis of the Rotational Translation Actuator (RTAC), an active control device which acts to suppress translational vibrations through controlled rotational motion. The vibration suppression arises from the rotational motion of an eccentric mass which is made to rotate and/or swing back and forth by the motor. The motor torque is proscribed by a controller board according to a feedback control law.

Sensors are used to measure the position of the eccentric mass and the linear acceleration of the machine to provide the feedback.

For comparison purposes, the vibrating machine is modeled as a linear mass-spring system with the RTAC rigidly mounted to the machine. The equations of motion for the device are then very similar to the classic 'rotating unbalance' problem (i.e., the unbalanced washing-machine) . Rather than a constant rotation rate exciting linear vibrations of the machine, however, the RTAC controls the rotation to reduce linear vibrations excited externally.

Although the machine vibration is linear, the eccentric mass motion introduces a nonlinearity into the dynamics. Moreover, this nonlinearity cannot be approximated as small if rotations and large oscillations of the eccentric mass are to be permitted. Thus, standard control strategies based upon classical linear feedback control theory cannot be applied. Our controller is based upon concepts from nonlinear control theory.

Two obvious advantages of the device are apparent. First, the device is not stroke-limited since the eccentric mass can rotate completely. Second, the device uses a widely available rotational motor for actuation.

Adaptive Dynamics of a Spring Supported Truss Member

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This paper examines an adaptive dynamical system in which system parameters are used as control parameters. The goal is ensure that one particular state is asymptotically stable. The effects of imperfections in this system are analyzed, and stabilities of the dynamical system are related to those of its associated static system. The particular system examined is isomorphic to the classical example of a ball rolling on a spinning hoop when the system parameters are held constant.

The simple adaptive structure is a rigid rod constrained to move in a plane by a pin support and torsional spring at its base. The torsional spring is undeformed when the rod is vertical, and the rod carries a load at its free end. This might be a member of a deployable truss for remote operation, say in space, which is to be launched in a folded state. For example, with the release of a catch, the compressed torsional spring drives the pin-free truss member with a load at the free end into the vertical position. Alternatively, this axially loaded torsion spring member might be part of a robot arm. The equations of this system are

$$
\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= -L(P + mg/2)\sin(x_1)/M + (k/M)x_1 + (c/M)x_2,
\end{aligned}
$$

where the state variable x_1 is the angle between the vertical and the rod, m is the mass of the rod, P is the load, c is the damping, L is the length, k is the spring constant, and $M = mL^2/3$.

The control objective is to recover from any rotation back to the vertical by modifying the system parameters. To control the positioning of the structure, or to act as a return mechanism for a deformed structure, the desired position must be made an asymptotically stable state by adjusting the control parameters. The possible control parameters are the applied load P, the damping c, the length L, and the spring parameters.

The static system is a cusp catastrophe if the spring is linear. The associated conservative dynamical system will then have a pitchfork bifurcation with respect to changes in the load. There are two zero eigenvalues at the critical point and these undergo splitting there. The load on the rod is assumed larger than the critical load corresponding to the static bifurcation point, $P_c = k/L$. In this case, the desired vertical position is initially statically unstable. To change the stability at a fixed load P, either the spring constant could be increased or a telescoping rod could be shortened.

The effect of a non-linear spring is analyzed by adding a quartic term to the spring potential energy to make the total potential energy of the system, V, a butterfly catastrophe. The butterfly is created if the coefficients of the quadratic and quartic terms in V are both zero at the same value of the control parameter, P. In the zero damping case, the stability of the steady states at a fixed load can be determined from the signs of the coefficients of the potential energy function. Making the spring relax as it deforms, so that the fourth order term in the Taylor series expansion of V is negative, destroys the stable steady states of the Hamiltonian pitchfork leaving just an unstable state at $(0,0)$, since the coefficient of x_1^6 in the expansion of V is negative. If the spring stiffens with deformation, then two stable states exist.

In the static system, it is possible to quasi-statically return a perfect conservative system to the vertical equilibrium state by adjusting these parameters, but is not possible if the system is imperfect. The imperfect system must have either a forcing function or damping to recover the desired position or one must be satisfied with the imperfect equilibrium state.

In the associated dynamical system, if the damping, c, is varied, a Hopf bifurcation occurs at $c = 0$. In this case, if the load is periodic, $P(t) = a\sin(\omega t) + P_o$, chaotic behavior can result if the ratio, a/c, of the amplitude to the damping is large enough.

As $k(t)$ or $L(t)$ vary continuously over time, no equilibrium state exists. For systems with more than one asymptotically stable state, the design strategy might be to vary system parameters to move the rod so that when the parameters are held constant at new values, the state of the rod lies in the basin of attraction of the desired asymptotically stable steady state.

The influence of initial imperfections in the undeformed position of the spring on the system behavior is also explored by examining a universal unfolding of the potential energy. Again, in this case, the vertical position will not be a steady state and one will have to be satisfied with a nearby state unless a forcing function is applied.

The Effects of Nonlinearities Upon Fuzzy Structural Control

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Timothy H-J. Yao[2]

Abstract

The area of structural control theory has matured considerably since being formalized by Yao [Yao, 1972] and applications of the theory exist for real-world problems. Within the area of active control strategies, most research has focused upon linear control [Soong, 1993]. While this is adequate for general applications, the unpredictability of natural hazards such as earthquakes and hurricanes makes it necessary to examine how active control strategies react within the nonlinear range of behavior.

The popular LQ control strategy (linear optimal control with quadratic cost function) has been shown to be deficient when systems move significantly into the nonlinear range of behavior. The instantaneous optimal control theory based on linear optimal control and developed by Yang et al. [Yang et al., 1988] might possess some stability problems [Spencer et al., 1992]. Suhardjo et al., apply an indicial formulation of nonlinear optimal control to Duffing oscillators [Suhardjo et al., 1992]. Other active control strategies include formulations in the frequency domain and application of neural networks.

Lotfi Zadeh first presented the concept of fuzzy set theory in [Zadeh, 1965]. Fuzzy set theory provides a mathematical structure for resolving imprecise or uncertain information that can be presented in fuzzy terms (usually by an expert). It is possible to extend fuzzy logic to the application of fuzzy control. In fuzzy control, system feedback is processed through stages of "fuzzification," resolution of the fuzzy rules, and "de-fuzzification" to yield the input to the control actuators.

Many researchers in the fields of information theory and automatic control have researched various aspects of fuzzy control. In a recent work, Jyh-Shing Roger Jang presents an example of a fuzzy inference system formulated on a neural network structure that can effectively predict nonlinear dynamic behavior [Jang, 1992]. This and other works have demonstrated the potential

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power of fuzzy control applied to nonlinear systems.

In this paper, the authors examine how fuzzy control may be applied to structural systems exhibiting nonlinear behavior. Nonlinearity is introduced in the form of mild hysteretic behavior. Emphasis is upon studying the effectiveness and robustness of the control of the nonlinear system. Sensitivity to parameter uncertainty is also explored.

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Robust Control for a Hybrid Nonlinear Partial Differential Equation

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ABSTRACT

The development of feedback controllers for nonlinear partial differential equations holds considerable promise as a practical method to modifying continuous systems in order to achieve a desired behavior. Several papers have appeared that demonstrate (experimentally and numerically) the feasibility of using feedback to control and stabilize finite dimensional chaotic systems. It is noteworthy that the lumped parameter systems investigated in [WSB] and [YYM] are obtained by using a Ritz discretization procedure to reduce the partial differential equation to a system of three ordinary differential equations that are similar to the Lorenz equations. This approximate model is then used to design a feedback controller that produces a steady non-oscillatory flow. Because of the special structure of the original nonlinear partial differential equation, the three mode lumped model decouples from the rest of the system and this feature makes it possible to use state feedback to control the nonlinear system. In particular, the use of the discretized model to design the controller for the continuous system works in this case because the important nonlinearity is captured by the three mode approximate model. Although this "approximate-then-design" method is a commonly used approach to such problems, it can lead to erroneous results and one must exercise care to ensure that the resulting design is robust.

Recently, Nayfeh, Nayfeh and Mook [NNM] gave a simple example of a nonlinear distributed parameter (continuous) system with the property that standard discretized lumped models fail to capture the essential nonlinear behavior of the dynamical system governed by the partial differential equation. This example also illustrates that an "approximate-then-design" approach to feedback control of distributed parameter systems

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can produce non-robust control laws. In addition, any practical feedback controller designed for a distributed parameter system must incorporate some type of state estimator and, regardless of the approach, one must introduce approximations at some point in the analysis.

In this paper we use distributed parameter control theory to design a robust feedback controller for the system given in [NNM]. The feedback law is linear, but the observer is based on a low order nonlinear model so that the controller is nonlinear. This approach allows us to construct a rigorous approximation of the full infinite dimensional feedback gain operator and hence obtain a practical low dimensional robust controller. We consider the hybrid system

$$\text{(1)} \qquad \rho \frac{\partial^2}{\partial t^2} w(t,x) = \frac{\partial}{\partial x}\left[\tau \frac{\partial}{\partial x} w(t,x) + \gamma \frac{\partial^2}{\partial t \partial x} w(t,x)\right] + \rho u_0(t,x), \quad 0 < x < 1, \quad t > 0$$

$$\text{(2)} \qquad m \frac{\partial^2}{\partial t^2} w(t,1) = -\left[\tau \frac{\partial}{\partial x} w(t,1) + \gamma \frac{\partial^2}{\partial t \partial x} w(t,1)\right] - \alpha_1 w(t,1) - \alpha_2 [w(t,1)]^2 - \alpha_3 [w(t,1)]^3 + f(t) + u_1(t)$$

with boundary condition

$$\text{(3)} \qquad w(t,0) = 0 .$$

Here, f(t) is viewed as a disturbance, $u_0(t,x)$ and $u_1(t)$ are control inputs and we have assumed strong internal damping. We use minmax theory for distributed parameter control to design a robust state feedback control law and combined this control law with an extended nonlinear observer to complete the design. The infinite dimensional Riccati operator is approximated by a convergent finite element scheme, yielding a finite dimensional controller. We present the results of several numerical experiments and illustrate how one can use this approach to address questions regarding optimal placement of actuators and sensors for robust control of structures.

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DIRECT OPTIMAL NONLINEAR CONTROL OF THE DUFFING DYNAMICS

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The study of dynamical systems via differential equations of motion is referred to as an "indirect method." On the other hand, the study of dynamics systems without any resort to or without any knowledge of differential equations of motion is referred to as the "direct method." In the direct method, algebraic equations of motion (AEM) take the place of the traditional differential equations of motion. The AEM are obtained by using Hamilton's Law of Varying Action (HLVA) in conjunction with the assumed-time-modes expansions of the generalized coordinates (Ref. 1). The constant unknown coefficients of the assumed-basis functions in time of these expansions become the generalized (algebraic) states of the dynamic system. If there are control inputs on the dynamic system, they too can be expanded in terms of assumed-basis functions in time multiplied by constant unknown coefficients of expansion playing the role of generalized (algebraic) control inputs.

By virtue of the assumed-time-modes expansions of the generalized coordinates and the controls, the variational work-energy quantities in HLVA can be integrated apriori in time over any time interval. This provides a set of purely algebraic equations describing the motion in terms of the constant unknown algebraic states and the algebraic control inputs for the time interval considered. The motion over arbitrarily large time intervals can be studied by a simple time-marching process invoking continuity conditions for the path of the motion. Below are some pertinent formulae of the direct method of control.

The HLVA is

$$\int_{t_0}^{t_1}(\delta T - \delta U + \delta W)dt - \frac{\partial T}{\partial \dot{q}}\delta q\Big|_{t_0}^{t_1} = 0 \qquad (1)$$

where T, U, W are the kinetic energy, potential energy and work functionals and q is an n-dimensional generalized coordinates vector. If f denotes an m-dimensional physical localized control inputs vector, the assumed-time-modes expansions over (t_0, t_1) for q and f are:

$$q = A_0(t)x_0 + A(t)\alpha \qquad f = B(t)\beta \qquad (2)$$

where x_0 is the set of known total initial states for q, $x_0 = \{q_r(t_0)\ \dot{q}_r(t_0)\}$ and α and β are unknown generalized algebraic states and the algebraic control inputs, respectively. A (t) and B(t) are matrices of admissible assumed basis functions in time over (t_0, t_1). Introducing the expansions (2) into HLVA, Eq. (1) for the energy functionals and carrying out the variations over to α, for any time-variant, time-invariant linear or nonlinear dynamic system one obtains the general form of the algebraic equations of motion:

$$P(x_0)\alpha + P_0(x_0)x_0 + N(x_0,\alpha) + Q(x_0,\alpha)\beta = 0 \qquad (3)$$

where the terms associated with N arise because of nonlinearities in the system.

To control the nonlinear AEM, optimally one can consider the quadratic regulator performance function

$$2J = \int_{t_0}^{t_1} (x^T W_x x + f^T R f)dt \qquad x = \left[q^T \dot{q}^T\right]^T \ ; \ W_x \geq 0 \ , \ R > 0 \tag{4}$$

Introducing the assumed-time modes expansions ((2)) into Eq. ((4)) one obtains the associated quadratic algebraic performance measure

$$2J = t_1\left[\alpha^T(D + V)\alpha + 2\alpha^T(D_0 + V_0)x_0 + \beta^T F\beta\right] + T(x_0) \tag{5}$$

where D, V and F are known algebraic weighting matrices arising from integrations of assumed-time-modes with the weighting matrices W_x and R. The minimization of (5) subject to the AEM (3) can now be carried out in a standard fashion, α and β being the unknowns. The general form of the explicit nonlinear control law is obtained in the form (Ref. 2)

$$\beta^* = G(\alpha)\alpha + G_0(\alpha)x_0$$
$$G = F^{-1}Q^T\left[Z_N{}^T - P^T\right]^{-1}(D + V)\ ; \ \ G_0 = F^{-1}Q^T\left[Z_N{}^T - P^T\right]^{-1}(D_0 + V_0) \tag{6}$$

Z_N is a Jacobian matrix of nonlinearities. To our knowledge, with Eq. (6), for the first time an explicit optimal nonlinear control law has been obtained for nonlinear systems.

In the paper, we illustrate the direct optimal control methodology outlined above for the Duffing dynamics. The explicit procedure for obtaining the AEM (3) for systems with quadratic, cubic, quartic etc... energy and work functions in generalized coordinates (hence in algebraic states) has been presented in Ref. (1). We specifically address the AEM formulation for the general form of the forced, damped Duffing Dynamics. Similarly, the explicit control law can readily be adapted to the Duffing dynamics.

In the paper, we shall also present various cases of the control of the single degree of freedom Duffing dynamics with different initial conditions, control weightings and set point regulations to illustrate characteristics of control of nonlinear systems. Because we are able to derive explicit nonlinear optimal control laws by the direct method, we are not hampered by the intricate numerical procedures as encountered by the Two Point boundary Value Problems of differential equations approaches. This allows us to better focus on the qualitative aspects of the nonlinear control problems without such distractions.

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Monday, June 13

0830-1010

Session 3. Modal Interactions

BISPECTRAL ANALYSIS OF NONLINEAR INTERACTIONS

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Nonlinear interactions play an important role in the dynamics of many systems (e.g., [1,2]). In this study, bispectral characterization of nonlinear interactions in a set of quadratically coupled oscillators is considered. These oscillators are governed by

$$\ddot{u}_1 + 2\mu_1\dot{u}_1 + {\omega_1}^2 u_1 + \alpha_1 u_1 u_2 = 0$$

$$\ddot{u}_2 + 2\mu_2\dot{u}_2 + {\omega_2}^2 u_2 + \alpha_2 {u_1}^2 = F cos(\Omega t) \tag{1}$$

where the frequencies ω_1, ω_2, and Ω are such that

$$\omega_2 = 2\omega_1 + \sigma_1; \quad \Omega = \omega_j + \sigma_2 \tag{2}$$

The quadratic phase coupling between the two modes of oscillation is studied as a function of the parameters α_1, α_2, and σ_1 by using bispectral analysis (e.g., [3-5]). This analysis allows us to characterize the interactions for weak as well as strong nonlinearities. Furthermore, the variation of phase coupling with respect to a control parameter (F or Ω) is also examined to ascertain the changes that take place near bifurcation points.

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EXPERIMENTAL INVESTIGATION OF RANDOM EXCITATION OF COUPLED BEAMS SYSTEM WITH MULTIPLE INTERNAL RESONANCES

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ABSTRACT

This paper presents the results of random excitation tests conducted on a nonlinear four-degree-of-freedom model. The model is designed such that it possesses multiple internal resonances. The system is excited by a band limited random process whose band width exceeds the highest natural frequency. The excitation and response signals are processed to evaluate statistical parameters such as spectral density functions, mean squares and probability density functions. The results are qualitatively compared with those predicted by the Monte Carlo simulation. The effects of nonlinear coupling parameters, internal detuning ratios and excitation spectral density level are considered in both results. It is found that both studies reveal similar dynamic features such as bifurcation points in the mean square response and modal interaction in the neighborhood of internal resonance. The experimental observation revealed that above certain excitation level, the response of the model becomes very large such that a possible catastrophe may take place. The numerical simulation revealed numerical instability at similar excitation levels.

NONSTATIONARY OSCILLATIONS OF AN ORBITING STRING
IN CONDITIONS OF MULTIPLE INTERNAL RESONANCES

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The transversal large-amplitude oscillations of an orbiting string satellite system can be suitably studied by two coupled partial integro-differential equations in the two transversal displacement components. The nonlinearities, mostly due to gyroscopic forces, are quadratic and quite small. So, the problem is weakly nonlinear and the only interesting nonlinear phenomena are those associated with internal resonances.

The string satellite system has a frequency spectrum similar to that of a taut string, apart from the first in-plane and out-of-plane pendulum type modes. On account of the different boundary conditions, in the description of transversal oscillations of this system cubic terms practically vanish while quadratic terms originating from gyroscopic forces are present. Although the nonlinearities are small in comparison with cables or taut strings, the presence of quadratic nonlinearities and the sequence of natural frequencies produce conditions of multiple internal resonances, which make this problem attractive to study, beyond its technical interest associated with the use of a satellite tethered to the orbiting shuttle in the space research.

As is well-known, when quadratic nonlinearities are present a high number of internal resonance conditions, primary and secondary, can occur. For the continuum system under study all these conditions are verified. In a general motion a high number of modes with amplitudes of the same order can therefore be involved. However, depending on the values of system characteristics and initial conditions, the motion can be again accurately described by a few modes. To this scope it is very important to select those modes among them the energy transfer is significant. For example, the in-plane and out-of-plane motions with a prevailing component are adequately represented by two modes, the prevailing one and the companion mode of frequency twice, forced by the quadratic terms. The stability analysis of the motion requires to add other modes. One mode is sufficient to describe the out-of-plane disturbance of the planar motion, while at least two new modes are necessary to investigate the stability of non-planar oscillations.

The multiple time scales method is used to obtain the equations that govern amplitude and phase modulations. The study of the planar resonant motion has already been completed. Fixed

points of the five first-order differential equations are evaluated. For increasing levels of energy, fundamental (two-mode solutions) and bifurcated (three-mode solutions) paths are determined for the case of the new component in primary or secondary resonance with the prevailing ones.

Nonstationary periodically amplitude modulated oscillations are analysed by numerically solving the three first-order differential equations. The trajectories are represented in the space of the state variables and lie on a torus.

The stability analysis of the three-mode constant amplitude solutions can be straightforwardly performed by means of the variational equations by determining the eigenvalues of the Jacobian matrix evaluated at each fixed point. The stability of the two-mode solutions is not straightforward because in this case the amplitude equations cannot be written in normal form. This difficulty can be overcome by using Cartesian representation of complex amplitudes. Here, however, a more general procedure has followed which is able to bring the stability analysis of periodically modulated solutions back to the analysis of variational equations with periodic coefficients. That is obtained by a suitable transformation of the complex amplitudes.

For the unstable two-mode planar oscillations the numerical solution of the five-dimensional system of the amplitude equations furnishes a description of the non-planar nonstationary motions in conditions of simultaneous resonances. Steady two-mode planar solutions bifurcate in periodic non-planar oscillations where the amplitudes of the involved three modes are slowly modulated with an energy transfer among in-plane and out-of-plane modes. When unstable planar periodic amplitude solutions are perturbed out-of-plane, the amplitude modulations of the planar components remain practically unchanged, but they occur around a mean value which is modulated on a slower time scale, similar to the case of unstable steady oscillations.

The investigation on the evolution of non-planar two-mode oscillations which involves four or more modes is now under study. The first results put into light a wider class of dynamic phenomena and give information on the approximation of discrete models for continuum systems obtained by a truncated mode series.

On the Hardware Development of an Internal Resonance Controller

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CANADA

This work focuses on the design and the fabrication of the electronic hardware used to implement a new control law based on the Internal Resonance (IR) phenomenon. The IR control strategy consists of introducing a second order supplementary system, or controller, to the plant, such that the plant and the controller are coupled through nonlinearities. The technique has been subjected to extensive investigation through simulation, and the research is focused now on implementing it. One possible scenario of application is to use a digital computer where the supplementary system is modelled by a discretized differential equation which needs to be solved, in real time, via software. Due to the complexity of the controller, the success of this method is dependent on the processing speed of the computer. Such a scenario can be costly and very difficult to implement.

In this paper, the development of an alternate implementation method is presented. The technique is hardware based. The supplementary system is developed by resorting to electronic components, and the integration algorithm is executed by analog circuitry. This technique possesses several advantages. Firstly, it eliminates any difficulties imposed by the limitations of the sampling and processing rates of digital circuitry. Secondly, it enables the application of simple real time control, where the feedback and controlling signals are continuous. Finally, it allows for the controller to be incorporated into a 'box' where the controller parameters can be easily modified to accommodate various plant characteristics and design criteria. The effectiveness of the hardware IR controller is investigated by regulating the oscillations of one and two DOF plants created with electronic components. The performance of the 'box' is in agreement with the results obtained by numerical simulation.

Multimodal Resonances of Elastic Structures

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1. Introduction

Multimodal resonances occur in elastic structures in various forms. Here are presented some typical multimodal resonances whose occurrence has been shown theoretically and experimentally.

2. Strings

A string subjected to harmonic excitation for large amplitudes is considered. The linearized natural frequencies of a string are in the ratio of 1 : 2 : 3 : ···. Due to this, multimodal resonances are expected to occur. It is shown that, near the second primary resonance point, the multimodal resonances occur in the form of fractional harmonic pair of (1/2, 3/2) type. Similarly it is shown that, near the third and fourth resonance points, the multimodal resonances occur in the form of fractional harmonic pairs of (1/3, 2/3) type and (1/4, 2/4, 3/4) type, respectively. In Fig.1 is shown an example of a wave obtained experimentally of the fractional pair of (1/4, 2/4, 3/4) type.

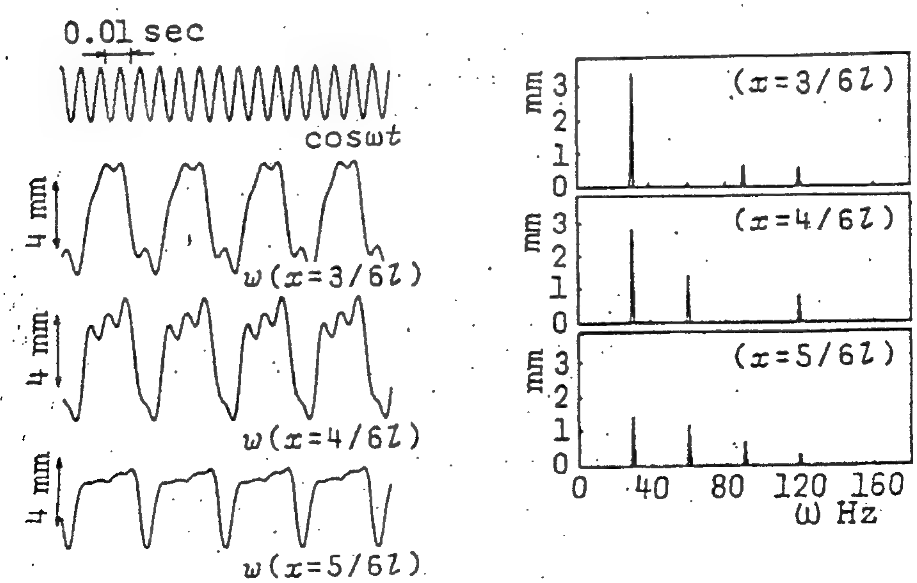

Fig. 1 Fractional harmonic pair of (1/4, 2/4, 3/4) type

3. Circular membranes

A circular membrane in axisymmetric problems subjected to harmonic excitation for large amplitudes is considered. The linearized natural frequencies of a circular membrane are in an approximate arithmetical progression. Due to this, multimodal resonances are expected to occur. It is shown that, near the second and third primary resonance points, the 3–modal and 5–modal resonances occur, respectively. In Fig.2 is shown an example of a wave obtained experimentally in the 3–modal resonances.

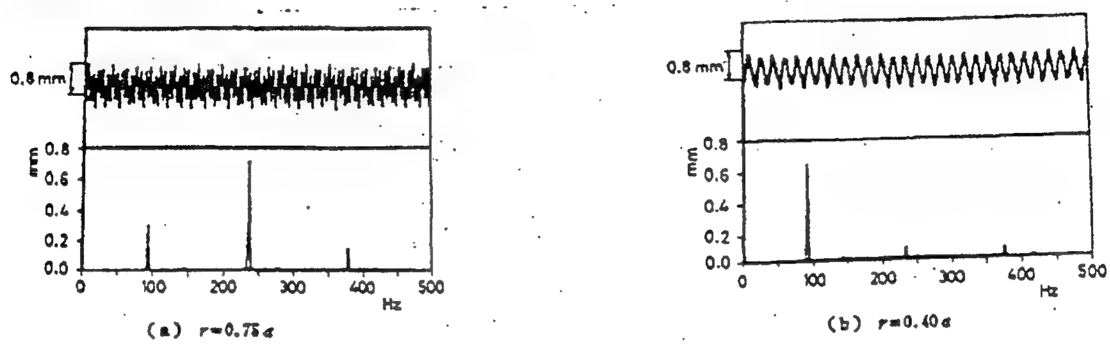

Fig. 2 Three–modal resonances

4. Square membranes

A square membrane subjected to harmonic excitation for large amplitudes is considered. In a square membrane, two modes exist in pairs with the same natural frequency and with the same modal shape. Due to this, multimodal resonances are expected to occur. It is shown that, near the primary resonance point having one nodal line, the modes in pairs are excited simultaneously with the phase lag of nearly $\pi/2$, and thus multimodal resonances occur in the form of a traveling wave. In Fig.3 is shown an example of a wave obtained experimentally of a traveling wave.

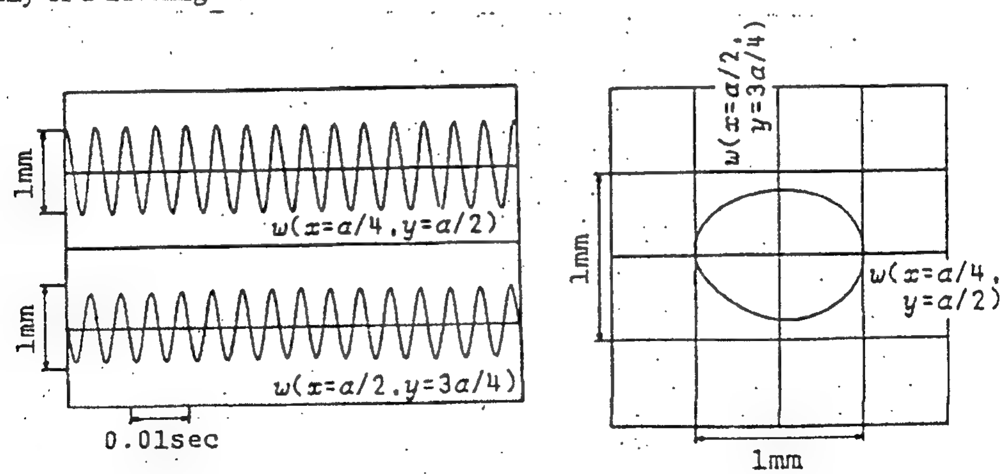

Fig.3 Traveling wave

Monday, June 13

1030-1210

Session 4. Parametric Vibrations

THE INFLUENCE OF PARAMETRIC VIBRATIONS ON NONLINEAR OSCILLATORS AND THEIR CONTROL

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Nonlinear oscillators can display a variety of different kinds of response behaviors, such as (stable) fixed points, periodic solutions (limit cycles), or complex (chaotic) behavior. If the oscillator equations depend on a (bifurcation) parameter, the bifurcation behavior can include the standard codimension 1 scenarios, as well as global bifurcation schemes. Under parametric (deterministic or stochastic) vibrations the response behavior will depend on the excitation frequency, the excitation range, and the excitation dynamics. Besides the analysis of the system response under the various influences mentioned above, a crucial problem is that of controlling the system response under parametric vibrations with the goal of e.g. stabilizing the system, or confining the response to a certain set below critical thresholds. This paper deals with a set-up, in which at least some of these problems can be formulated and analyzed.

Consider a nonlinear oscillator given by an ordinary differential equation

$$\dot{x} = f_0(x;p) \quad \text{in } \mathbb{R}^d. \tag{1}$$

Here $p \in \mathbb{R}^n$ denotes a vector of system parameters, which can be structure parameters (i.e. fixed for a given system), design parameters (i.e. they can be chosen within given

bounds to achieve a desired system performance), perturbed parameters (i.e. subject to deterministic or stochastic excitation), or controlled parameters (i.e. they can be affected by input signals in an open loop or feedback fashion). Taking into account the different kinds of system parameters, we rewrite (1) as

$$\dot{x} = f_0(x;\alpha) + \sum_{i=1}^{m} \xi_i(t) f_i(x;\alpha) + \sum_{j=1}^{\ell} u_i(t) g_i(x;\alpha), \tag{2}$$

where $\alpha \in I \subset \mathbb{R}$ is a design parameter, $\{\xi_i, i = 1 \dots m\}$ describes the perturbation with dynamics $f_1 \dots f_m$, and $\{u_i, i = 1 \dots \ell\}$ describes the control inputs with dynamics $g_1 \dots g_\ell$. A large body of literature is available on the (bifurcation) analysis of the unperturbed, uncontrolled system $\dot{x} = f_0(x;\alpha)$. Attention in the mechanics literature has focused for a while on the perturbed, uncontrolled system, where the perturbation is given by its dynamics, its frequency (often under the additional assumption of small perturbations), or by white noise, leading in this case to the qualitative analysis of stochastic differential equations.

Recent progress in the mathematical theory of skew product flows (perturbation flows and stochastic flows) allows us to analyze the perturbed, uncontrolled system

$$\dot{x} = f_0(x;\alpha) + \sum_{i=1}^{m} \xi_i(t) f_i(x;\alpha) \tag{3}$$

in a unified way for deterministic and stochastic bounded excitations $\{\xi_i, i = 1 \dots m\}$ with values in $U^\rho \subset \mathbb{R}^m$, where $U^\rho = \rho \cdot U, \rho \geq 0$, with U compact convex and $0 \in int\ U$. The system (3) has two bifurcation parameters, $\alpha \in I$ and $\rho \geq 0$, and the interplay between the system and the perturbation dynamics as well as the value of α and the perturbation range ρ determine the response behavior of the system. It turns out that, if α_0 is not a bifurcation point of the nominal system $\dot{x} = f_0(x;\alpha)$, then there is a perturbation range $\rho \in [0, \rho_0)$ such that the perturbed system (3) has basically the same behavior as the nominal system. This statement can be made precise in terms of Morse sets of flows and their ordering. If, however, α_0 is a bifurcation point of the nominal system, then arbitrarily small perturbations can lead to a drastically different behavior of (3). Various examples, including the Van der Pol-oscillator, the Takens-Bogdanoff oscillator,

and dynamics of chemical reactors will be given to illustrate the theory. Furthermore, for higher dimensional systems, we will analyze stochastic perturbations of chaotic systems and the phenomenon of 'transient chaos'.

Controlling a nonlinear oscillator with parametric vibrations is one of the major design tasks in nonlinear control theory. Robust stabilization under parametric, external, or dynamic uncertainties is quite well understood for linear systems, using H^∞-theory, metrics in functions spaces, or Lyapunov exponents. For nonlinear systems of the type (2) we present a Lyapunov exponent approach to this problem that yields complete results for single-degree-of-freedom systems. The idea is to linearize the perturbed and controlled nonlinear oscillator about the fixed point and to control the Lyapunov exponents of the linearized system via an associated dynamical system on the projective space. This leads to precise robust stabilization criteria locally around the fixed point. A combination with the methods developed to analyze globally the perturbed system (2) as described above, gives information on the stabilizability region of the nonlinear system (3). It should be pointed out that the stabilizing feedback laws will, in general, be nonlinear and discontinuous. The example of the Van der Pol oscillator will be studied in detail.

Another goal of controlling a nonlinear oscillator with parametric vibrations is to keep the system response within a certain set, below critical thresholds. In addition to the theory explained for the response analysis of (3) above, one needs here the characterization of multistability regions, from which the response of the perturbed system will approach various, distinct limit sets. In many cases the problem then boils down to the control of the size of the multistability regions. Results on the bifurcation behavior of these regions are given in terms of bifurcation analysis of control sets, and numerical methods to determine the corresponding control sets and multistability regions are available. The theory and its applications to the control of nonlinear, perturbed oscillators are illustrated with various examples of one-degree-of-freedom systems.

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On a parametrically Excited Extensible Pendulum

D.J. Shippy and H. Fu

ABSTRACT

This study is concerned with the motions of a extensible pendulum whose support is displaced sinusoidally along a vertical line. The system is governed by two coupled nonlinear ordinary differential equations. The two normalized dependent variables in the equations represent the angular motion of the pendulum and the axial motion of the mass particle, which is connected to the support through a linear spring.

The extensible pendulum has been studied by many researchers, especially for the free vibration case. The interesting behavior of energy transfer between the two modes at the internal resonance and the conditions for instability have been demonstrated and discussed. While the chaotic motion of the pendulum also has been investigated, only the conservative, free-vibration case has been considered and for just a few selected value of system parameters.

The present study considers an extensible pendulum with parametric excitation and damping. The purpose is to investigate systematically the solution bifurcations and the routes to chaos with variation of system parameters. We have fixed some parameter values so that the internal resonance always exists and have allowed two other parameters, forcing frequency and forcing amplitude, to vary in a significant portion of that parameter plane. The ranges of these parameters are such that the forcing frequency range includes two external resonances, namely fundamental resonance and principal resonance with respect to the angular motion, and the forcing amplitude range covers both small angular oscillations and monotonic rotational motion. Then we have varied the value of the damping parameter to examine its effect on the motion in the entire parameter plane. For the region of small system response, a perturbation method is carried out about the two resonant frequencies to obtain approximate analytical solutions. Then the stability boundaries were determined. The analytical results are compared with numerical simulations. For the region of large system response, numerical integration solely is used.

The main result is an extensive parameter diagram of forcing amplitude versus forcing frequency, showing the regions of various types of motions and the boundaries of stability. Since the axial oscillation always exists, all motions observed can be characterized according to four general categories, based upon the behavior of the corresponding angular motion:

(a) no angular motion, which occurs for small forcing amplitude but loses stability as the forcing amplitude increases;
(b) oscillatory motion of various periods, in which the direction of rotation changes with time and for some parameter values the amplitude of angular motion even exceeds π;
(c) monotonic rotation (always in a single direction) with periodic velocities of various

periods;

(d) chaotic motion in both modes.

The overlapping, bifurcations and transitions of these motions make the parameter diagram complicated and interesting. For the principal-resonant forcing frequency and a particular damping value, a typical evolution of system response is observed as the following. The system has trivial angular motion for small forcing amplitude. When the forcing amplitude increases to a critical value, the trivial motion becomes unstable, and a motion with period-1 axial response and period-2 angular response takes place. As forcing amplitude becomes larger, the transient modulations of both axial and angular amplitudes exist longer before they converge to steady-state motion. This converging process can last for thousands of forcing cycles until the forcing amplitude reaches a value for which the response becomes chaotic. If the forcing amplitude is increased further, at least seven other periodic motions with various periods can be observed in addition to chaos. Some of them are large amplitude oscillations with angular amplitude exceeding π. Others are monotonic rotations interspersed with intermittent oscillations. On the other hand, when the forcing frequency is close to the fundamental resonance, as the forcing amplitude increases the trivial angular motion at first bifurcates into either an angular oscillation or a monotonic rotation, depending on initial conditions. Then the motion becomes chaotic for larger forcing amplitude. With further increase in the forcing amplitude, a large amplitude, period-2 axial oscillation and period-4 angular oscillation appear.

We have found that the routes to chaos of this system include crisis and subharmonics and also are related to the existence of the so-called recurrence phenomenon, the energy transfer between the two modes. As the forcing amplitude increases, the transient modulations of response amplitudes due to the transfer of energy exist longer and longer and lead to chaos for some parameter values. This unique route is one of our main interests in the study. Since the damping has a great effect on the recurrence, its variation can change the route to chaos qualitatively.

For some particular parameter values, there exist multiple attractors in the phase space. The steady-state response of system will depend on the initial conditions. Demonstrating this dependence globally for the four-dimensional phase space can be very time-consuming even with a powerful computer. Therefore, in our study we employ a numerical scheme based upon the concept of cell-mapping to create the charts of basin boundaries in phase space approximately.

ANALYSIS OF NONLINEAR TIME-PERIODIC DYNAMICAL SYSTEMS UNDER CRITICAL CONDITIONS

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Abstract: Development of methods such as center manifold reduction[1] and normal forms[2] in the time-dependent domain for the analysis of nonlinear time-periodic dynamical systems under critical conditions are presented. In the past, particularly when the systems are autonomous[3] such methods have been found extremely useful in the study of dynamics of critical systems. For time-periodic systems a straight forward generalization of such techniques is not possible due to the fact that the linear part of the variational equation is time-dependent. However, this difficulty can be overcome by the use of the Liapunov-Floquet (L-F) transformation matrix. Some recent computational procedures developed in references [4-6] indicate that it is possible to obtain a representation of the state transition matrix (STM) of linear periodic systems in terms of Chebyshev polynomials which is suitable for algebraic manipulations[7]. Application of Floquet theory and the eigen analysis of the STM at the end of one principal period provides the L-F transformation matrix in terms of the Chebyshev polynomials[8]. Since this is a periodic matrix, a Fourier representation of the elements is found to be convenient for the present analysis. It is well-known[9] that such a transformation converts a linear periodic system into a linear time-invariant one! When applied to quasilinear equations with periodic coefficients, a dynamically similar system is obtained whose linear part is time-invariant and the nonlinear part consists of coefficients which are periodic. Due to this property of the L-F transformation, a periodic orbit in original coordinates will have a fixed point representation in the transformed coordinates.

In this study, the dynamics of the critical, quasilinear, time-periodic equations, obtained after the application of the L-F transformation, is studied by employing *time-dependent center manifold reduction* and *normal form theories*. Two physical examples, namely, a parametrically excited simple pendulum and a double inverted pendulum subjected to non-conservative periodic loadings are considered. Stability of these systems under critical conditions are studied using the local methods developed here and Poincaré maps obtained numerically. The three generic codimension one bifurcations namely, Hopf, flip and fold bifurcations are analyzed. It is found that the system characters have been captured by the local methods as well as the numerical Poincaré maps, correctly. It is to be noted that the technique presented herein is applicable even to those systems where the linear part does not contain either a small parameter or a time-invariant component at all. Furthermore, secondary bifurcations and stability of time-periodic systems have also been explored by combining harmonic balance and the L-F Transformation methods. Bifurcation diagrams for the example problems have been generated and compared to the diagrams shown by Flashner and Hsu[10].

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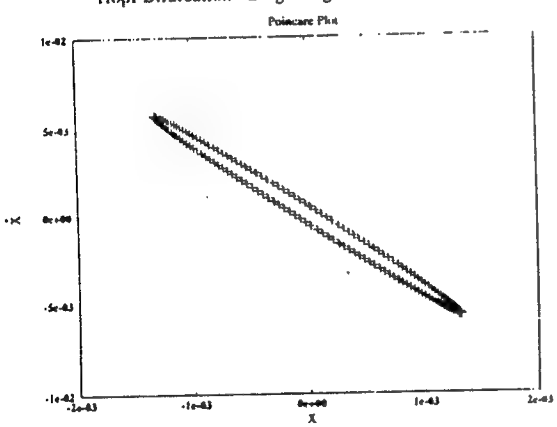
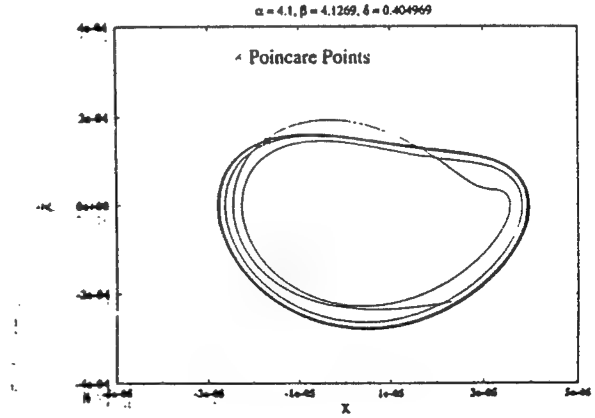
Hopf Bifurcation - Single degree of freedom system



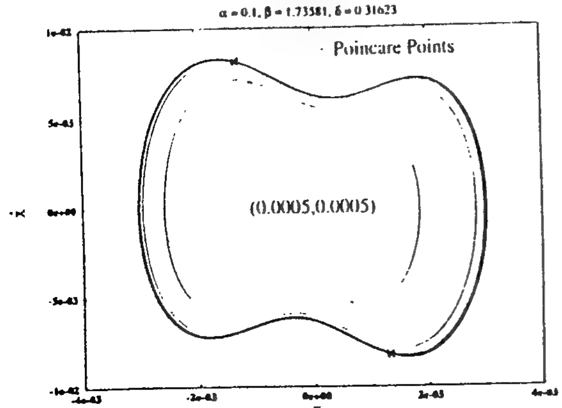
Fold Bifurcation - Single Degree of Freedom System

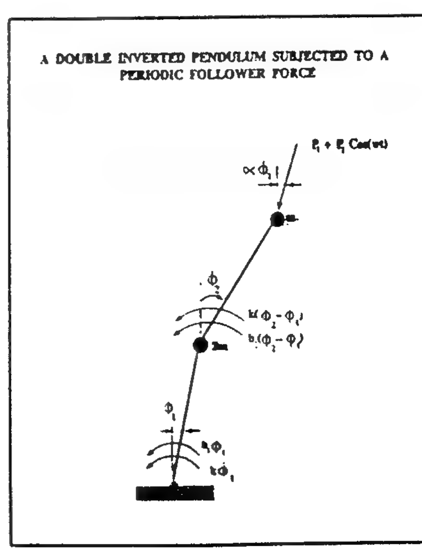
$\alpha = 4.1, \beta = 4.1269, \delta = 0.404969$



Flip Bifurcation - Single degree of freedom System

$\alpha = 0.1, \beta = 1.73581, \delta = 0.31623$





NONLINEAR ANALYSIS OF SUBHARMONIC PARAMETRIC RESONANCES OF A CANTILEVERED PIPE CONVEYING FLUID

by

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Abstract

This paper deals with the nonlinear dynamics and stability of cantilevered pipes conveying fluid, where the fluid has a harmonic component of flow velocity superposed on a constant mean value. The mean flow velocity is near the critical flow velocity for which the pipe becomes unstable by flutter through a Hopf bifurcation, and the pulsations in the flow are assumed to be small. The forcing frequency is approximately equal to twice that of the limit cycle. The eight-dimensional (four-degree-of-freedom) system is reduced to a simpler set of equations using the method of averaging, yielding the stability conditions for subharmonic parametric resonances.

Stability maps are then constructed in terms of three main parameters: the frequency and the velocity detuning and the amplitude of the perturbation. Using a fourth-order Runge-Kutta integration scheme, the complete set of equations are also integrated, for a wide range of parameters. The analytical solutions are compared with those obtained numerically, and some interesting results are presented. It is shown, both analytically and numerically, that periodic and quasiperiodic oscillations can exist, depending on the parameters.

Finally, these results are also compared with some preliminary experiments undertaken in our laboratory, utilizing elastomer pipes conveying water. The pulsating component of the flow is generated by a plunger pump, and the motions are monitored by a noncontacting optical follower system.

THEORETICAL AND EXPERIMENTAL INVESTIGATION OF THE RESPONSE OF INITIALLY CURVED RECTANGULAR PLATES

by

Sadok SASSI and Germain L. OSTIGUY

ABSTRACT

Extensive efforts have been devoted over the past decades to the investigation of static and dynamic behavior of thin walled structures containing initial geometric irregularities (imperfections). A survey of the literature reveals that a considerable amount of research has been concentrated on the theoretical and experimental prediction of the natural frequencies. However, one can notice that less attention has been relatively devoted to the investigation of possible effects of imperfections on the interaction between different resonances arising from parametric excitation.

In the frame of the present work, we analyze, theoretically and experimentally, the effect of initial curvatures on the response of rectangular plates subjected to parametric loading and bring new clarifications about the problem of nonlinear modal interaction in plates with initial geometric imperfections and containing more than one kind of resonances.

The plate analyzed is subjected to the action of periodic in-plane forces uniformly distributed along two opposite edges. Four sets of boundary conditions are considered: (1) all edges simply supported, (2) loaded edges simply supported and the two others loosely clamped, (3) loaded edges loosely clamped and the two others simply supported, (4) all edges loosely clamped.

The nonlinear plate theory used in this analysis is the dynamic analog of the Von karman's large-deflection theory modified to take into account the initial imperfections and derived in terms of the stress function **F**, the lateral displacement **W** and the imperfection W_0.

The problem consists in determining the right functions F and W which can satisfy

the differential equations and the boundary conditions. The solutions for these functions are represented by a double series consisting of the appropriate beam eigenfunctions that satisfy the boundary conditions. Applying the Galerkin method to the governing equations, using the orthogonality properties of the assumed functions and performing numerous calculations leads to a system of nonlinear ordinary differential equations for the time.

The temporal response is analyzed by the first-order generalized asymptotic method and the proposed solution for the temporal equations of motion takes into account the possibility of existence of simultaneous forced and parametric vibrations.

The results obtained when solving the eigenvalue problem, indicate that the presence of imperfections of the order of a fraction of the plate thickness may significantly raise the resonance frequencies. Such effect depends strongly on the type of boundary conditions, the mode shape of the imperfection and the mode shape of the vibration. Moreover, the results show interesting features concerning "avoided crossing" and "coalescence", a characteristic of eigenvalue problems where there is coupling between modes.

The analytical results of the investigation reveals that, contrarily to the case of perfect plates, the parametric and combination resonances arising from the excitation of imperfect plates are possible for any kind of boundary conditions.

The theoretical and experimental results confirmed that, if two different types of resonances may occur independently for the same excitation frequency, an interaction between them is highly probable. Particularly, when a forced resonance region overlaps a parametric resonance one, the interaction of the two mechanisms manifests itself in different ways. In order to gain further insight into various aspects of this problem and to clarify the nature of the interaction mechanism, the Overlapping Factor (*OF*) and the Positioning Factor (*PF*) have been introduced. Using those two parameters, the following conclusions could be driven: The nature and the importance of any interaction depend strongly on the loading conditions, the relatif position and the degree of overlap between the relevant instability zones and the type of imperfection (mode and amplitude).

Monday, June 13

1330-1510

Session 5. Impact and Friction

DYNAMIC ANALYSIS OF ROTARY-AUGMENTER DEVICE AND OPTIMUM DESIGN OF RUBBER DAMPER OF BLAST HOLE DRILLS MACHINE

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Jin Youxin

(Shenyang Industry University , P.R. China)

Abstract

The Blast Hole Drills cause strong vibration during their working process. It is necessary to analyze the danamic properties of the Blast Hole Drills in order to reach high drilling efficiency, reduce the machines' vibration and prolong the lifetime of the machine and the cones. The paper makes some analysis in the longitudinal and rotary vibration of rotary loading system of the Blast Hole Drills. A reasonable dynamical model is set up. The main vibration characteristics of the system was gotten through the calculation.

Multiple Impacts with Friction
in Rigid Multibody Systems

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Abstract: In practical applications nonlinear dynamics often arise from impacts and stick slip phenomena. Typical examples are walking machines, frictional dampers between turbine blades, assembly processes of manipulators and impact dynamics in gear boxes. Systems of that type have a time variant structure and show a nonsmooth behaviour due to friction and impacts. This requires a special treatment for the analytical and numerical solution procedures. In the following an impact model based on the hypothesis of Poisson and Coulomb is described which is completely compatible to unilateral constrained motion with Coulomb friction.

One important property of unilaterally constrained systems is that their number of degrees of freedom varies with time, which results from different states of contacts between the rigid bodies of the mechanical system. In detail each of the possible contacts may show sliding, stiction or separation and these contact situations may occur in any combination. The dynamic system reaches its maximal number of degrees of freedom f at the condition that all contacts show separation. This state is used to describe the system by a set of generalized coordinates $\boldsymbol{q} \in \mathbb{R}^f$. Each of the possible contact constraints is controlled by kinematic indicators like relative distance g_N and relative velocity in normal and tangential direction $\dot{g}_N$ and $\dot{g}_T$. A normal or tangential contact constraint is said to be potentially active if the necessary conditions for contact, $g_N = \dot{g}_N = 0$, or stiction, $g_N = \dot{g}_N = \dot{g}_T = 0$, are fulfilled. A normal or tangential constraint is said to be active if in addition the relative acceleration is equal to zero, $\ddot{g}_N = 0$ or $\ddot{g}_T = 0$. Each active constraint reduces the number of degrees of freedom by one. Thus the constraints on the acceleration level are taken into account as algebraic secondary conditions and are included in the equations of motion as additional contact forces λ_N or λ_T. If the set of active constraints is known, a representation is achieved where an index 1 system can be integrated. Therefore the last step is the determination of the active set.

Under the assumption of certain contact laws, such as a unilateral impenetrability condition in the normal and a Coulomb friction constraint in the tangential direction, the state transitions from stiction to sliding and from contact to separation can be handled by solving a Linear Complementarity Problem of the form $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$; $\boldsymbol{y} \geq 0$; $\boldsymbol{x} \geq 0$; $\boldsymbol{y}^T\boldsymbol{x} = 0$. The terms $\boldsymbol{y}$ and $\boldsymbol{x}$ contain the physically complementary magnitudes of the normal and tangential constraints which are in that case the relative accelerations $\ddot{\boldsymbol{g}}$ and linear combinations of the contact forces $\boldsymbol{\lambda}$. This formulation allows a simultaneous treatment of multiple state transitions occuring by several switching events at the same point of time or by induced transitions of coupled contact problems. A unique solution of the contact problem with respect to the contact forces $\boldsymbol{\lambda}$ exists if the matrix $\boldsymbol{A}$ of the LCP is

positive definite. It turns out, however, that the limited tangential forces of the sticking contacts and the Coulomb friction forces of the sliding contacts may destroy this structure and may lead to existence and uniqueness problems, related to "static" and "dynamic" friction. Here the first time impacts with friction occure: The areas of existing unique solutions in the dynamic friction problem are separated from those of nonexistence or nonuniqueness by an event that can be called "impact without collision". A formulation of the constraints on the velocity level allows the reduction of the dynamic friction problem to the static one by introducing contact impulses $\boldsymbol{\Lambda}$ instead of contact forces $\boldsymbol{\lambda}$.

For a complete description of all possible state transitions the events "separation to sliding", "separation to stiction" and "separation to separation" are still missing. They are treated by the concept of impacts with friction. Usually the impact in the normal direction is modelled by the hypothesises of Newton or Poisson. Both approaches make use of a coefficient of restitution ε, which is defined as the proportion of the relative velocities after and before the impact in the first case, $\varepsilon = -\dot{g}_{NE}/\dot{g}_{NA}$, and as the ratio of the normal impulses during expansion and compression in the second case, $\varepsilon = \Lambda_{NE}/\Lambda_{NC}$. Even in the frictionless case both concepts are different for multiple impacts. Generally Poisson's hypothesis allows an energy transfer between the normal and the tangential direction and therefore does not lead to the energy gaps which are observed by Newton's law when control parameters are changed. Thus a general purpose impact model based on Poisson's hypothesis is developed, where the absolute values of the tangential impulses are bounded by the frictional law of Coulomb. It is proven that this model is always dissipative or energy preserving and under certain conditions coincides with the results given by applying a time scalation method and using the Newton-Euler-equations during the impacts. It contains the special effect of the impacts without collisions and is applicable even if dependent constraints are present. It also includes the case that no impulses are transfered at non-zero approaching velocities, and can handle induced separations of existing contacts. Combined with the theory of unilaterally constrained motion locking effects in the static and dynamic sense can be treated.

All together the numerical solution procedure of nonsmooth mechanical systems can be regarded as a sequence of initial value problems of ordinary differential equation systems with varying dimensions, which are selected by the solution of complementarity problems. The nonsmooth behaviour of such systems is generated by state transitions from sliding to stiction on the acceleration level and by frictional impacts on the velocity level. Both types of unsteady events lead to similar complementarity conditions which are linear in the planar case and are solved by a complementary pivot algorithm. The theory is applied to some basic examples of systems with more than one contact point for illustrating the effects described above.

Nonlinear Elastic Dynamic Contact Problems In Travelling Wave Ultrasonic Motors

submitted to

Fifth Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures

to be presented by

Jörg Wallaschek
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Germany

Abstract:

In the travelling wave ultrasonic motor, electrical energy is transformed to high-frequency mechanical oscillation by means of piezoelectric elements. The stator of the motor oscillates in such a way that material points on the stator surface perform an elliptical motion. The rotor is pressed to the stator and is driven by frictional forces generated in the contact zone.

In this paper the nonlinear mixed boundary value problem of the stator/rotor contact is formulated and solved using three basic models for the frictional contact:

- Area contact and Coulomb friction assuming a stiff stator and a soft contact layer neglecting tangential deformation
- Area contact and Coulomb friction assuming a stiff stator and a soft contact layer including tangential deformation
- Area contact and Coulomb friction including the compliance of stator and rotor.

The results allow to estimate the most important motor characteristics like e. g. speed-torque curve, overall efficiency and stall torque density as a function of the motor parameters. Moreever optimal material parameters and the influence of different operating conditions can be determined. The numerical results are compared to experiments and the experimental results are used to identify the parameters of the motors used in the experiments.

The results described in the paper have been obtained in the framework of a research project funded by Deutsche Forschungsgemeinschaft under research grants DFG Wa 564/6-1 and DFG Wa 564/7-1.

Dynamics of Flexible Mechanical Systems with Contact-Impact and Plastic Deformations

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ABSTRACT

A computer based formulation for the analysis of rigid-flexible mechanical systems has been investigated as a feasible method to predict the impact response of complex structural systems.

A general methodology for the dynamic analysis of rigid flexible multibody systems is presented using the augmented lagrangian formulation. Component mode synthesis is used to reduce the number of flexible degrees of freedom. An algorithm is proposed using a joint coordinate approach.

In many impact situations, the individual structural members are overloaded giving rise to plastic deformations in highly localized regions, called plastic hinges. This concept is used by associating revolute non linear actuators with constitutive relations corresponding to the collapse behavior of the structural components.

A continuous force model based on the Hertz contact law with hysteresis damping is also included. The choice of model parameters is discussed.

The effect and importance of structural damping schemes in flexible bodies is also addressed.

The validity of this methodology is assessed by comparing the results of the proposed models with those obtained with an experimental test where a beam collides transversally with a rigid block. Another example of a torque box model impacting a rigid barrier, is presented.

Motions of a mass-spring system between two rigid asymmetric barriers, stability and unstability

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The work presented in this paper starts from the works of P. Holmes, G.X. Li, F.C. Moon, R.H. Rand, and S.W. Shaw, on impacting systems. We study the simple asymmetric system shown in Figure 1.

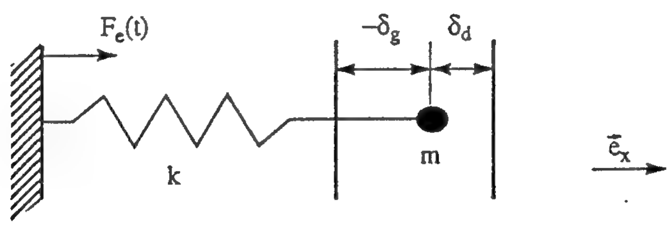

Figure 1: The physical system.

A mass m is attached to a linear spring of stiffness k. When the displacement x exceeds $-\delta_g$ or δ_d the mass m contacts a barrier. The system is externally excited by a harmonic force $F_e(t)$. In addition, we assume
1) that the coefficient of restitution r, which indicates how much energy is lost at impact, is constant ;
2) that the impact is instantaneous, and ;
3) that there is no damping during the free flight.

The non-dimensionalized equations of motion are as follows :

$$\left\{ \begin{array}{rcll} \ddot{x} + x & = & A_e \sin(\omega_e t) & -1/\rho < x < 1 \\ \dot{x} & \rightarrow & -r\,\dot{x} & x = -1/\rho \text{ or } 1 \end{array} \right. \tag{1}$$

where $0 < r < 1$ and $0 < \rho \leq 1$, $\rho = \delta_d/\delta_g$, $0 < \delta_d \leq \delta_g$.

The main goal of this work is to determinate the domains where the solution is periodic and stable, and those where the solution is unstable, in the parameters space (A_e, ω_e), r and ρ being fixed. In the first part, we focus on two types of periodic motions as shown in Figure 2.

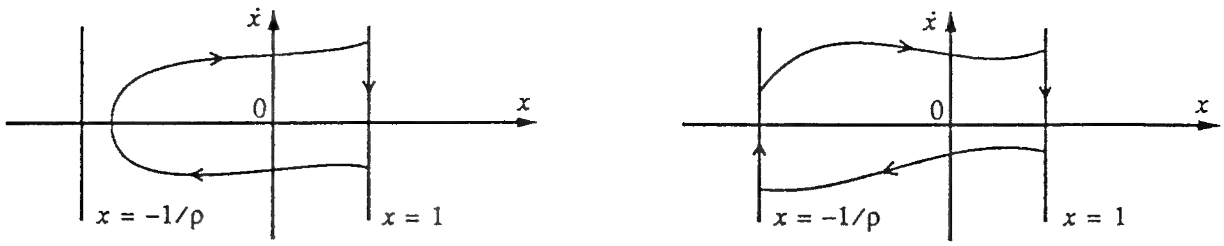

Figure 2: Types of periodic motion searched.

In each case the period of the orbit is $n\,T_e$, $n \in I\!N^*$. Then we study the stability of these periodic orbits. To start with, we build up the first return Poincaré map $\mathcal{P}$, and we establish that $\mathcal{P}$ is a C^∞ map from $\mathcal{V}^0 \subset \Sigma \to \Sigma$ in a neighbourhood of one of the periodic orbits shown in Figure 2. Σ is the cross section of Poincaré defined as

$$\Sigma = \left\{ (x, \dot{x}, t) \in I \times I\!R \times S \,\text{such that}\, x = +1,\, \dot{x} > 0 \right\},$$

where $I = [-1/\rho, 1]$ and $S = [t_0^0, t_0^0 + n\,T_e]$ *modulo* $n\,T_e$.

In addition, when the barriers are non-symmetrical about the central position of the mass, we show that the pitchfork bifurcation is impossible.

In the last part, we discuss the limit as the asymmetric system tends to the symmetric one, that is $\rho \to 1$. The main results are the following :

- when $n \in I\!N^*$ is even, the saddle-node and the period doubling bifurcations exist ; all periodic orbits are asymmetric ;
- when $n \in I\!N^*$ is odd, the saddle-node bifurcation of the case where the barriers are non-symmetrical about the equilibrium position of the mass, is changed into a pitchfork bifurcation at the limit $\rho = 1$; there are at least three types of bifurcation : saddle-node, pitchfork and period doubling, and there are two sorts of periodic orbits : symmetrical and non-symmetrical.

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Monday, June 13

1530-1710

Session 6. Rotor and Structural Dynamics

On the Counteraction of Periodic Torques in Rotating Systems By Means of Centrifugally Driven Vibration Absorbers

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A class of mechanical systems is considered which freely rotate about a fixed axis and are subjected to applied periodic torques. In this work we address the counteraction of these torques by means of centrifugally driven masses which move along prescribed paths relative to the rotating system. The linear theory for these centrifugal pendulum vibration absorbers has been well understood for many years and some work on nonlinear aspects of their dynamics has been carried out. However, past studies have focussed on the performance of absorbers with specific prescribed paths when the system is subjected to a given torque. Herein we provide a classification scheme to identify those torques which can be exactly counteracted by a single absorber or by a pair of identical absorbers moving out-of-phase with respect to each other. In addition, we provide a technique for generating the attendant absorber path(s) for these torques and demonstrate the method with several examples. Also, a design strategy is offered by which one can use sets of absorbers to cancel a general periodic torque. These results provide a basis for future work in which optimal paths can be designed for torques which cannot be exactly cancelled by a desired number of absorbers.

FORCED OSCILLATIONS OF A VERTICAL CONTINUOUS ROTOR WITH GEOMETRICAL NONLINEARITY

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1. INTRODUCTION

Nonlinear forced oscillations of a vertical continuous rotating shaft are discussed. The restoring force of the shaft has geometrical nonlinearity. The possibility of the occurrence of various nonlinear oscillations and their characteristics are investigated.

2. EQUATIONS OF MOTION AND NATURAL FREQUENCIES

A theoretical model is shown in Fig. 1. Both ends of the shaft are supported freely. The coordinate system O-xys is fixed in space and O-$\xi\eta s$ rotates with the shaft at an angular velocity ω. The components of an unbalance are represented by $e_\xi(s)$ and $e_\eta(s)$, respectively. The symbol c is the damping coefficient. Shear deformation is neglected. The deflections in Ox- and Oy-directions are denoted by $u(s,t)$ and $v(s,t)$, and a complex value $z=u+iv$ is defined by them. The equations of motion are given by

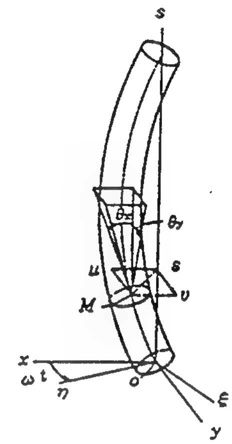

Fig. 1

$$\left.\begin{array}{l}\dfrac{1}{\pi^4}\dfrac{\partial^4 z}{\partial s^4}+\dfrac{\partial^2 z}{\partial t^2}-\dfrac{\kappa}{\pi^2}\left[\dfrac{\partial^4 z}{\partial s^2\partial t^2}-2i\omega\dfrac{\partial^3 z}{\partial s^2\partial t}\right]+c\dfrac{\partial z}{\partial t}\\ \quad -\dfrac{\alpha}{\pi^4}\dfrac{\partial^2 z}{\partial s^2}\displaystyle\int_0^1\left(\dfrac{\partial z}{\partial s}\right)\left(\dfrac{\partial \bar{z}}{\partial s}\right)ds=\omega^2\left[e_\xi(s)+ie_\eta(s)\right]e^{i\omega t}\end{array}\right\} \quad (1)$$

in dimensionless form, where $\bar{z}$ is a complex conjugate of z. Free oscillations of a corresponding linear system with no damping are expressed approximately by

$$z(s,t)=Z\sin n\pi s\cdot e^{ipt} \qquad (n=1,2,\cdots) \qquad (2)$$

The natural frquencies p are given by

$$G(p)\equiv\nu_n^4+2\kappa\nu_n^2\omega p-(1+\kappa\nu_n^2)p^2=0 \qquad (3)$$

The roots are shown in Fig. 2. Symbol p_{fn} (>0) has a forward whirling mode and p_{bn} (<0) has a backward mode.

The deflections are represented by summations of deflections of each modes as follows.

$$u=\sum_{n=1}^{\infty}u_n(t)\varphi_n(s)\,, \quad v=\sum_{n=1}^{\infty}v_n(t)\varphi_n(s) \qquad (4)$$

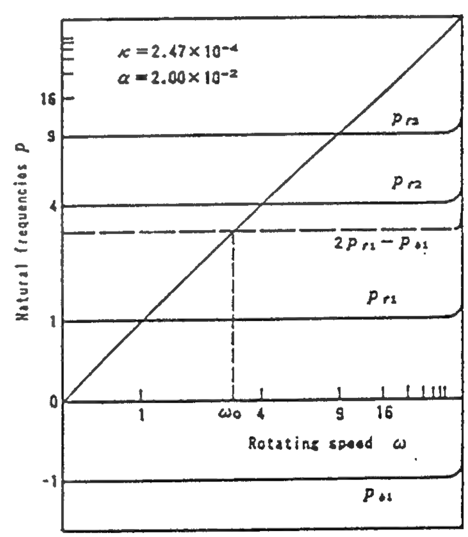

Fig. 2. $p-\omega$ diagram

By substituting Eq. (4) into Eq. (3), and using the orthogonality of modes, we get the following equations on u_n and v_n.

$$\left.\begin{array}{r}(1+\kappa\nu_n^2)\ddot{u}_n+c\dot{u}_n+2\kappa\omega\nu_n^2\dot{v}_n+\nu_n^4u_n+\alpha\nu_n^2u_n\displaystyle\sum_{j=1}^{\infty}\dfrac{\nu_j^2}{2}(u_j^2+v_j^2)\\ =\omega^2(a_n\cos\omega t-b_n\sin\omega t)\\ (1+\kappa\nu_n^2)\ddot{v}_n-2\kappa\omega\nu_n^2\dot{u}_n+c\dot{v}_n+\nu_n^4v_n+\alpha\nu_n^2v_n\displaystyle\sum_{j=1}^{\infty}\dfrac{\nu_j^2}{2}(u_j^2+v_j^2)\\ =\omega^2(a_n\sin\omega t+b_n\cos\omega t)\end{array}\right\} \quad (5)$$

where, $n=1, 2 \cdots$, and

$$e_\varepsilon(s)=\sum_{n=1}^{\infty} a_n \varphi_n(s) \ , \ e_\eta(s)=\sum_{n=1}^{\infty} b_n \varphi_n(s) \tag{6}$$

3. HARMONIC OSCILLATIONS [p_{fn}]

The oscillations [p_{fn}] at the major critical speed are represented approximately as follows. (The symbol[$*$] means that it appear when the relation $*=\omega$ holds.)

$$u_n = P\cos(\omega t+\beta),\ v_n = P\sin(\omega t+\beta) \tag{7}$$

By the harmonic balance method, we get

$$\left.\begin{aligned} 2\omega P\dot{\beta} &= G_n(\omega)P+(1/2)\alpha \nu_n^4 P \\ &\quad -\omega^2(a_n\cos\beta+b_n\sin\beta) \\ 2\omega\dot{P} &= -c\omega P-\omega^2(a_n\sin\beta-b_n\cos\beta) \end{aligned}\right\} \tag{8}$$

Resonance curves are shown in Fig. 3.

4. SUMMED-AND-DIFFERENTIAL HARMONIC OSCILLATION [$2p_{fm}-p_{bn}$]

In the summed-and-differential harmonic oscillation [$2p_{fm}-p_{bn}$], two components appear predominantly in addition to the harmonic component. Their frequencies are expressed approximately as follows.

$$\left.\begin{aligned} \dot{\theta}_{fm} &= \omega_{fm} \equiv (p_{fm0}/\omega_0)\omega \\ \dot{\theta}_{bn} &= \omega_{bn} \equiv (p_{bn0}/\omega_0)\omega \end{aligned}\right\} \tag{9}$$

where p_{fm0} and p_{bn0} are the values of p_{fm} and p_{bn} at ω_0. For the case $m=n$, the solution are expressed by

$$\left.\begin{aligned} u_n &= R_{fn}\cos(\omega_{fn}t+\delta_{fn})+R_{bn}\cos(\omega_{bn}t+\delta_{bn}) \\ &\quad +P_n\cos(\omega t+\beta_n) \\ v_n &= R_{fn}\sin(\omega_{fn}t+\delta_{fn})+R_{bn}\sin(\omega_{bn}t+\delta_{bn}) \\ &\quad +P_n\sin(\omega t+\beta_n) \end{aligned}\right\} \tag{10}$$

For the steady-state oscillation, we can get the resonance curves shown in Fig. 4.

For the case $m\neq n$, we get only the trivial solutions. This means that this type of oscillation does not appear.

5. SUMMARY OF THE THEORETICAL ANALYSIS

The results of the theoretical analysis are summerized in Table. 1. We can know that some specific kinds of oscillation appear in such a rotor system.

6. EXPERIMANTAL RESULTS

The experiments were performed for two shafts with 10mm and 4 mm in diameter. Both shafts have the same length of 800mm. In the speed range up to about 4500rpm, a harmonic oscillation and one kind of summed-and-differential harmonic oscillation were observed. The latter is shown in Fig. 5.

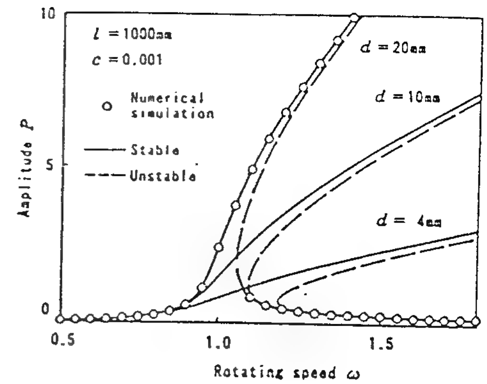

Fig. 3 Harmonic oscillatillation [p_{fn}]

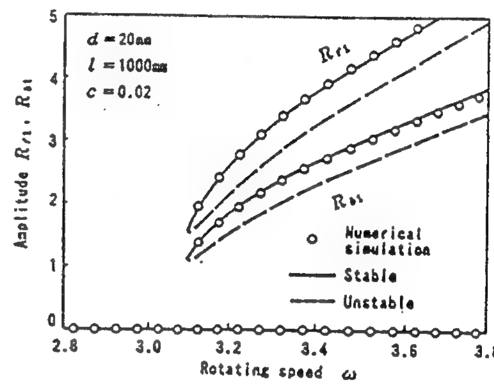

Fig. 4 Oscillatillation [$2p_{fm}-p_{bn}$]

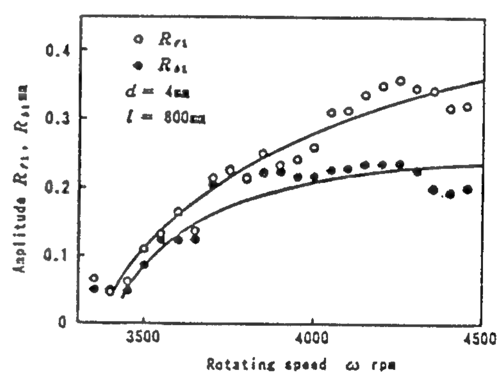

Fig. 5 Experimental results

	Kinds	Linear	Nonlinea
Harmonics	p_{fn}	Appear	Appear
Sub-harmonics	$2p_{fm}$		×
	$-2p_{bm}$		×
	$3p_{fm}$		×
	$-3p_{bm}$		×
Summed-and-differential harmonics	$p_{fm}-p_{bn}$		×
	$p_{fm}+p_{fn}$		×
	$-p_{bm}-p_{bn}$		×
	$2p_{fm}-p_{bn}$		○ $m=n$
	$p_{fm}-2p_{bn}$		×
	$2p_{fm}+p_{fn}$		×
	$-2p_{bm}-p_{bn}$		×
	$p_{fl}+p_{fm}+p_{fn}$		×
	$p_{fl}+p_{fm}-p_{bn}$		○ $l=n$
	$p_{fl}-p_{bm}-p_{bn}$		×
	$-p_{bl}-p_{bm}-p_{bn}$		×

Table 1.

Nonlinear Response of Rotors Supported on Journal Bearings

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Abstract

Sub-synchronous whirling of journal bearing supported rotors (oil whirl) is known to be a potentially damaging mechanism in industrial rotor-bearing systems. Previous analytical studies on oil whirl were done on *long* journal bearings [Myers, 1984, Shaw and Shaw, 1990]. These studies showed that whirl instability is a result of Hopf bifurcation in a balanced rotor and periodically perturbed Hopf bifurcation in the case of rotors with very small level of imbalance. In this paper, threshold speeds of instability and imbalance response of rotors supported on *short* journal bearings, which are widely used in industry, is studied analytically and numerically. First, analytical criteria for oil whirl (Hopf bifurcation) for a balanced rotor is derived following Myers's analysis for *long* bearings. Both subcritical and supercritical regimes of oil whirl are identified. Then, for an unbalanced rotor, bifurcations of periodic response are studied numerically over practical speed ranges and levels of imbalance. Sub-harmonic, quasi-periodic, and chaotic motions of the rotor are predicted. A shooting algorithm with a pseudo arc-length continuation scheme is used for locating periodic solutions, calculating their stability, and continuation through bifurcation points. Finally, the overall behavior of the rotor-bearing system is presented through global bifurcation diagrams.

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Myers, C.J., 1984, "Bifurcation Theory Applied to Oil Whirl in Plain Cylindrical Journal Bearings", *Journal of Applied Mechanics*, **51**: 244-250

Shaw, J., and Shaw, S.W., 1990, "The Effects of Unbalance on Oil Whirl", *Nonlinear Dynamics*, **1**: 293-311

NONLINEAR DYNAMICS OF ROTOR SYSTEMS EXPERIENCING RUBBING

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This study is concerned with investigating the nonlinear dynamics of rotor systems with and without a laterally flexible disk while experiencing rub. A rotating, flexible continuous disk/shaft model was developed and the dynamical behavior of this system with and without rubbing was studied. The model developed in this study is similar to the Jeffcott rotor model except that the disk is treated as a laterally flexible continuous circular plate. The motion of the disk was transformed from physical coordinates to a set of generalized coordinates under which the generalized motion was uncoupled and the responses were calculated. Then the inertial moment acting on the shaft was computed and introduced into the governing equations of the shaft motion.

The equation development for this model is based on the following considerations:

(1) The disk is flexible only in the lateral direction in which the motion is the most significant. In other directions it is rigid.

(2) The rotational motion of the rotor is not coupled with its translational motion. Only rotational vibration is considered. This is because only the rotational motion is coupled with the disk flexibility.

(3) The rotor speed is constant during rubbing.

(4) The disk and the supports of the rotor are symmetrical.

(5) The mass of seal/casing is neglected. Because the shaft/disk/blade is more flexible than the seal/casing, the deformation of seal/casing is usually small or only a small part of the mass of seal/casing participates in the motion. The rub force is mainly dependent on the elasticity of the system in most cases.

After some mathematical manipulations, the dimensionless equations of motion for the

whole system can be obtained as

$$\ddot{x}_1 + r_1\dot{x}_2 + \sum r_{2i}(\ddot{x}_{3i} + 2\dot{x}_{4i}) + 2\xi_1\frac{1}{\lambda_1}\dot{x}_1 + \frac{1}{\lambda_1^2}x_1 + f_{n1} = \overline{F}\cos\tau \qquad (a)$$

$$\ddot{x}_2 - r_1\dot{x}_1 + \sum r_{2i}(\ddot{x}_{4i} - 2\dot{x}_{3i}) + 2\xi_1\frac{1}{\lambda_1}\dot{x}_2 + \frac{1}{\lambda_1^2}x_2 + f_{n2} = \overline{F}\sin\tau \qquad (b)$$

$$\ddot{x}_{3i} + 2\dot{x}_{4i} + 2\xi_{3i}\frac{1}{\lambda_{3i}}\dot{x}_{3i} + (\frac{1}{\lambda_{3i}^2} - 1)x_{3i} = -(\ddot{x}_1 + 2\dot{x}_2) \qquad (c)$$

$$\ddot{x}_{4i} - 2\dot{x}_{3i} + 2\xi_{3i}\frac{1}{\lambda_{3i}}\dot{x}_{4i} + (\frac{1}{\lambda_{3i}^2} - 1)x_{4i} = -(\ddot{x}_2 - 2\dot{x}_1) \qquad (d)$$

$$i = 1, \ldots$$

In the above equations, x_1, x_2, x_{3i} and x_{4i} are scaled by e, the imbalance eccentricity. Time is scaled by the rotor speed, Ω. The f_{n1} and f_{n2} are nonlinear forces caused by rub.

Direct integration and the harmonic balance method were used to study the steady state motion of the system. A number of parameter variation studies were performed for varied rub clearances and disk mass influence ratios. By using orbit plot, Poincare map and other techniques, The system responses to the rub, its occurrence and development, and the global stability of the observed responses were studied. Periodic, quasiperiodic and chaotic motions and multi-valued responses were all observed in this study.

The results show that rub can be classified into two types: light rub and heavy rub, and the light rub has the forms of forward, backward, or mixed whirling motion. The results also show that the disk flexibility may alter the critical speed to some degree and may also significantly affect the amplitude and stability of the rotor vibration.

An experimental work was performed to verify the behavior of a rotor system experiencing rubbing, as predicted in previous analytical studies. The rotor rigs used in the experiments were designed to have included disk flexibility and a rubbing mechanism. The governing equations of motion are similar to those studied in analytical investigations. The development of rub responses, rotor orbit trajectories, and multi-valued responses (light rub and heavy rub) were experimentally studied. Shaft vertical and horizontal displacements and key-phasor signals are sent to the analyzer. Spectrum and orbit plots were recorded and plotted. The results show the rubbing response development from light forward bouncing, mixed forward bouncing, to high amplitude backward whirling heavy rubbing. These agree very well with the results from an earlier analytical study.

Invariant Manifolds, Nonlinear Vibrations in a Singularly Perturbed Nonlinear Oscillator with Applications to Structural Dynamics

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Summary: We are interested in the dynamics of viscoelastic nonlinear structural systems comprised of substructures with diverse stiffnesses. In particular, we seek relations of the dynamics of a structural system to the dynamics of a simpler system obtained in the limit when some of its stiff substructure become perfectly rigid. We are motivated by the fact that simple structures such as rods, beams, plates etc. are supported by other simple structures (supports) with sufficiently larger stiffnesses. For instance, consider a structural system composed of a rotating shaft and its mounts, a rotating propeller and its shaft, a plate with enbedded sensors and actuators for active control etc. All these structural systems are composed of substructures with stiff substructures (mounts, shaft, and actuating material) and soft substructures (rotating shaft, propeller blades, and plate). The fundamental problem to be solved is to determine how the dynamics of a structural system is related to the dynamics of a simpler structural system obtained in the limit as the stiff substructures become infinitely stiff, that is rigid. Then once this relation has been established, it is again fundamental to delineate how the qualitative dynamics evolve as the magnitude of the stiffness of the stiff substructure becomes of the same order of the magnitude of the stiffness of the soft substructure.

In this paper we study the dynamics of a nonlinear oscillator coupled naturally to a linear oscillator. The linear stiffness of the nonlinear oscillator is negative and the nonlinearity is cubic. This coupled system of oscillators captures the qualitative dynamics of a buckled nonlinear beam with its ends hinged on two vertical columns. This geometrically idealized structural system is a representative of the collection of all structural systems with the property that they contain stiff substructures.

When the stiffnesses (frequencies) of the two coupled oscillators are sufficiently apart, their motion evolves in two distinct time scales: One time scale, called fast time, is related to the stiff linear oscillator; the other time scale, called slow time scale, is related to the more compliant nonlinear oscillator. As the stiffnesses approach each other, the time scales cannot be distinguished so easily. We describe the motions of this coupled system of oscillators in the context of invariant manifolds. By viewing the equations of motion as a singular perturbation of the nonlinear oscillator (the singular parameter being the ratio of the linear frequency of the nonlinear oscillator to the frequency of the linear oscillator), and using the theory of center manifolds, we have shown that there exists an invariant manifold of motion. This is called a slow invariant manifold since it carries motions evolving in slow time scale. The slow

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manifold is nonlinear, smooth and 2-dimensional. All motions carried by it are such that the nonlinear oscillator slaves the linear oscillator into slow and in-phase motions. Furthermore, the slow invariant manifold is an equilibrium for the fast component of the motion. The projection of the slow invariant manifold onto the phase space of the nonlinear oscillator defines the phase space of a nonlinear oscillator. This nonlinear oscillator, called the slow reduced system, is a regular perturbation of the Hamiltonian nonlinear oscillator. And it describes the long time motions such as nonlinear oscillations and chaotic oscillations of the forced dissipative coupled. We have computed various orders of approximation of the slow invariant manifold and the corresponding reduced system. Numerical experiments show that the long time motions of the dissipative and forced dissipative coupled oscillators are very close to the motions of the approximate reduced system. There exists also a linear invariant manifold with the property that all motions carried by it are such that the linear oscillator slaves the nonlinear oscillator into out-of-phase fast motion. Viewing the motions of the coupled oscillators with respect to the slow invariant manifold reveals the remarkable result that the fast components of the motion is described by an unforced, uncoupled linear oscillator, although the linear oscillator is forced. All motions enter a neighborhood of the slow invariant manifold since it is attractive. The dynamics in this neighborhood are described as follows: The free uncoupled oscillator forces the reduced system until the motion is captured by the slow manifold. Once on the slow invariant manifold, the motion evolves only in slow time. The slow motion of the nonlinear oscillator is described by the reduced system, and that of the linear oscillator by the function satisfying the manifold condition.

Tuesday, June 14

0830-1010

Session 7. Computational Methods

A COMPARISON OF THE GLOBAL CONVERGENCE CHARACTERISTICS OF SOME FIXED POINT METHODS

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Abstract

Since locating all the fixed points of a nonlinear oscillator involves the numerical solution of simultaneous equations, it is useful to observe some of the global convergence characteristics of these techniques. Specifically, the popular Newton or quasi-Newton approaches require numerical evaluation of the Jacobian matrix of the Poincaré map. This study focuses attention on the domains of attraction for a number of fixed point techniques applied to a single nonlinear oscillator with a single set of parameters. Clearly, there are many issues here, including proximity to bifurcations, order of the dynamical system, temporal convergence characteristics, i.e. CPU time, and so on, but it is instructive to observe a snapshot of the basins of attraction which underlie path-following routines, when a parameter is changed.

Introduction

Locating fixed points of a dynamical system often reduces to the computational solution of sets of nonlinear simultaneous equations. This type of problem is ubiquitous in applied mathematics and finds application in chemical engineering [Lucia et al. 1990], electric circuits [Aprille and Trick, 1972], and applied mechanics [Bishop et al. 1988]. A thorough exposition of the various techniques for the numerical solution of nonlinear equations can be found in Dennis and Schnabel [1983]. More recent progress within the framework of dynamical systems theory is contained in Parker and Chua [1989], and software is available for path-following based on continuation [Doedel, 1981].

The study outlined in this note compares a number of solution techniques based on Newton's method in terms of their global convergence, or transient, characteristics. Specifically, the numerical evaluation of the Jacobian matrix of the Poincaré map is assessed using a grid of initial conditions. Since these methods are discrete dynamical systems themselves, it is not surprising that rather complicated domains of attraction result when all the fixed points (both stable and unstable) of a particular differential equation are evaluated.

We shall focus on the periodic solutions of a single, dissipative, periodically driven, nonlinear oscillator of the form

$$\dot{\vec{x}} = f(\vec{x}, t) \tag{1}$$

where $\vec{x}$ is a two-dimensional vector containing position and velocity and $f(\vec{x}, t) = f(\vec{x}, t + T)$, and the parameters of the equation are held fixed. The forcing phase may be used to trigger a Poincaré section resulting in a discrete map of the form

$$\vec{x}_{i+1} = P(\vec{x}_i) \tag{2}$$

where the 3-D flow of Equation 1 has now been reduced to the 2-D map of Equation 2. Fixed points of the Poincaré map correspond to the roots of the residual map defined by

$$G(\vec{x}) = P(\vec{x}) - \vec{x} = 0. \tag{3}$$

This system can be solved using Newton's method:

$$\vec{x}_{k+1} = \vec{x}_k - (J - I)^{-1} G(\vec{x}_k) \tag{4}$$

where the (2 x 2) Jacobian matrix, J, is the matrix of partial derivatives of P. It is the numerical evaluation of J that is the subject of consideration in this note.

Specifically, the methods of first-order finite differencing and integration of the variational equation are applied to finding the roots of a residual map corresponding to the periodic solutions of a typical ordinary differential equation. The effects of incorporating a line search and Broyden updating are included. In contrast to most studies on convergence characteristics, spatial, rather than temporal or CPU time, behavior is highlighted. Although this is just a "snapshot" of typical behavior, this type of subtle dependence on initial guesses may have relevance in path-following routines where the location of a fixed point may be changing rapidly under a smooth variation in a system parameter.

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EFFICIENT SIMULATION OF SYSTEMS WITH DISCONTINUITIES AND TIME-VARYING TOPOLOGY

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Summary

The numerical simulation of the behaviour of dynamical mechanical systems does not give rise to difficulties if the forces remain smooth and the number of degrees of freedom does not change during the time of simulation. In mechanical engineering, however, some kinds of forces are modelled in a way that allows some form of non-smoothness, as is the case for dry friction forces according to the Amontons-Coulomb law or internal forces in material plasticity models, or the velocities may change discontinuously due to impulsive forces at impacts. Furthermore, the number of degrees of freedom may change, for instance if a gripper of a robotic manipulator seizes an object or lift-off of some part occurs. The use of normalization techniques and standard integration methods may cause large computation times or inadequate results. This is especially so if many points of non-smoothness are present in the solution or if some important qualitative characteristic of the solution depends on the occurrence of a discontinuity.

The considerations of the preceding paragraph urge the development of integration methods that can efficiently deal with points of non-smoothness. The methods that are proposed here are based on integrating the equations of motion until some point of non-smoothness is reached and restarting with possibly adjusted initial values and equations after this point. Generally, the points of non-smoothness depend on the solution that is being calculated and are not known beforehand. These points of non-smoothness can be characterized by the vanishing of some test function or indicator function. If a zero is detected within some time step, the solution in this time step is approximated by an interpolation formula, which eliminates the need for further evaluations of the accelerations. The location of the zero of the test function is then obtained by a root finding method. This method is based on the same ideas as the method of Brent but is in a form that is more appropriate for the present application. The present method can handle the case that more than one zero exists in the time step, which can easily be overlooked by other methods.

As an example of application, the crash behaviour of a railway train is studied. For the purpose of design and optimization of the crash behaviour, simple models with a limited number of degrees of freedom are used for an assessment of the overall behaviour. These simple models allow a far more efficient simulation than detailed finite element or finite difference models that are used in so-called explicit codes, so many cases can be studied. The models have nearly all sources of non-smoothness that may occur in mechanical systems: contact and impact, fracture, dry friction and material plasticity.

With the proposed models and simulation procedures, analytic sensitivity analyses with respect to design variables and initial values can be made, which offers the possibility to apply existing optimization techniques.

NONLINEAR STRUCTURAL RESPONSE USING ADAPTIVE DYNAMIC RELAXATION ON A MASSIVELY-PARALLEL-PROCESSING SYSTEM

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Use of the finite element method to solve structural problems of increasing computational size and complexity continues to be the focus of intense research. Yet even on current high-speed vector computers, solution costs, especially for transient dynamic analyses, are often prohibitive. Emerging high-performance computers offer tremendous speedup potential for these types of applications, provided an optimal solution strategy is implemented. Existing sequential solution procedures may be adapted to operate on these computers. However, these procedures have been developed and customized for sequential operation and may not be the best approach for parallel processing. To exploit this potential fully, problem formulations and solution strategies need to be re-evaluated in light of their suitability for parallel and vector processing. As such, the overall goal of this research is to develop an adaptive algorithm for predicting static and dynamic response of nonlinear hyperelastic structures which exploits these emerging high-performance computing systems.

The basic formulation for the adaptive dynamic relaxation (ADR) algorithm for hyperelastic structures is given by Oakley and Knight in an earlier work. Dynamic relaxation is a technique by which the static solution is obtained by determining the steady-state response to the transient dynamic analysis for an autonomous system. In this case, the transient part of the solution is not of interest, only the steady-state response is desired. Since the transient solution is not desired, fictitious mass and damping matrices which no longer represent the physical system are chosen to accelerate the determination of the steady-state response. These matrices are redefined (using existing equations) so as to produce the most rapid convergence. For highly nonlinear problems where stiffness changes significantly during the analysis, adaptive techniques exist which automatically update the integration parameters when necessary.

An ADR algorithm represents a unified approach for both static and transient dynamic analyses, and is known to be very competitive for certain problems with high nonlinearities and instabilities. Reliability is ensured by integration parameters which are adaptively changed throughout an analysis to accommodate Although a very small time step is generally required to ensure numerical stability, the computational cost per time step is very low and is mostly associated with evaluation of the internal force vector.

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The present paper builds on a study which was begun to develop efficient single-processor and multiprocessor implementations of the ADR algorithm and evaluate their performance for the static analysis of nonlinear, hyperelastic systems involving frictionless contact. For problems of this nature, the ADR method may represent one of the best approaches for parallel processing. Performance evaluations on single-processor computers have shown that ADR is reliable and highly vectorizable, and that it is competitive with direct solution methods for the highly nonlinear problems considered. In contrast to direct solution methods, it has minimal memory requirements, is easily parallelizable, and is scalable to more processors. It also avoids the ill-conditioning related convergence problems of other iterative methods for nonlinear problems. The objective of the present paper is to evaluate the performance of a massively parallel implementation of ADR.

A parallel ADR algorithm is developed for nonlinear structural analysis and implemented on the 512-processor Intel Touchstone DELTA system. It is designed such that each processor executes the complete sequential algorithm on a subset of elements. One-dimensional strip partitioning and two-dimensional block partitioning are used to divide the problem domain among the available processors. Load balancing is ensured by the use of structured, uni-material meshes. Efficient schemes are developed to accomplish the required nearest-neighbor and global communication. The parallel algorithm is used to solve for the nonlinear static response of 2-D and 3-D cantilever beam problems and 3-D arch, tunnel, and torus problems.

Correctness of the parallel algorithm is verified by running all test cases to completion on the DELTA. Final results are consistent with those obtained using a single-processor. Completion times for the large 3-D test cases are minimal and demonstrate both the computing power of the DELTA and the ability of the ADR algorithm to fully exploit this power. Moreover, the current multiprocessor implementation is not vectorized. A vectorized version should lead to further increases in performance. The minimal memory requirements of ADR are again demonstrated as the largest test case runs successfully on a single DELTA processor equipped with 16 Mbytes of memory.

Impressive speedups are achieved using the DELTA, especially for the large 3-D models. This performance may be attributed to the minimal interprocessor communication ADR requires relative to computations and the efficient schemes with which this communication is accomplished. These speedup results demonstrate the high scalability of the ADR algorithm and show that the algorithm can be implemented on at least 512 processors without significant performance degradations. Thus, ADR provides the potential for efficiently exploiting large numbers of processors to substantially reduce the solution time of highly nonlinear problems. In this context, ADR represents a very promising approach for parallel-vector processing.

An Iterative Scheme of Point Mapping Under Cell Reference for Global Analysis

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Abstract

It is well accepted that numerical computation is extreme importance in modern nonlinear analysis. In fact the ready availability of extensive computational resources that has led to the current resurgence of interest in nonlinear system behavior, much of which now centers on global system characteristics. Unfortunately, the cost of numerical simulations of systems by point mapping method are extremely expensive.

In analyzing the inefficiency of point mapping method in global analysis, authors find that every initial points have to be led to reside on certain steady state solutions in point mapping method and, thus a steady state solution is always unnecessarily calculated hundreds and thousands of times. Point mapping method has been carried out on basis of long term determination of dyamical characters of an initial point, thus every trajectories starting from initial points are extended until they reside on steady state solutions. Undoubtfully, the method implemented in this way is accurate but is uneconomic. In fact, some important things in global analysis are neglected: 1) points in a trajectory have same global properties as the initial point, that is, all points in a trajectory are in a same basin of attraction, thus they could be used to depict the basin of attraction; 2) the density of trajectories or processed points, whose global property are known, is nonuniformly distributed in a chosen region of the state space even if a uniformly distributed initial points are used, for instance, the densities of processed points in the subregions around attractors are much higher than other subregions. This implies that global characteristies of some subregions may be determined only with small

number of uniformly distributed initial points being used. After global characters of some subregions have been determined, there is no need to set new initial points in these subregions. Above all, all these accurately determined subregions can play the role of attractors which a trajectory could be led to in finite time rather than " infinite time", computational works could be reduced greatly.

To take the advartages of these characteristics in numerical global analysis, we propose an iterative scheme of point mapping under cell reference (iterative PMUCR) in this paper. After establishing a cell coordinate system in state space as reference to divid chosen region into subregions, we can define the attracting sets composed by the subregions whose global properties could be determined by applying point mapping method on small number of initial points, then an iterative procedure could be performed repeatly to determine fine resolution of basins of attraction of a nonlinear dynamical system.

In contrast to previous methods which use approximate mapping to reduce the the computational work, for instance, cell—to—cell mapping by Hsu [1,2] and interpolated cell mapping by Tongue and Gu [3], iterative PMUCR uses accurate mapping, that is, point mapping determined by numerical integration. As mapping used in present method is accurate and a trajectory is terminated whenever it comes into the areas of attracting sets which takes finite time, present method could achieve exactly same results as those of point mapping method but reduces the computational cost to a great degree. Morever, iterative PMUCR is self — examinatable method, that is, one could easily know the correctness of the results by checking if any attracting cells in the attracting sets, used as targets of trajectories in an iterative procedure, change to boundary cells when the computation of this iterative procedure is finished.

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PANEL FLUTTER STUDIES IN HYPERSONIC FLOW BASED ON THE NAVIER STOKES EQUATIONS

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ABSTRACT

Addressed in this paper is the problem of a panel fluttering in a hypersonic stream. We treat this as an aero/thermoelastic problem in the sense that in addition to aeroelastic coupling we also account for thermal coupling between the fluid and structure.

More specifically we have formulated the panel flutter problem in hypersonic flow based on the Navier-Stokes equations, and studied the influence of the surrounding temperature field and the heat transfer between the fluid and panel on the panel's aeroelastic behavior. The evolution equations are analysed numerically and qualitatively. Attention has been paid to bifurcations which occur as control parameters in the system vary. Long-time histories, phase-plane plots, power spectra of the responses, and Lyapunov exponents of the responses are the tools used in studying the system under consideration.

Tuesday, June 14

1030-1210

Session 8. Bifurcations

Bifurcations in Planar Piecewise Linear Systems

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In this study, we consider the following class of nonlinear dynamical systems on $\Re^2$,

$$\dot{x} = \begin{cases} A_1(x - x_{e1}) & \text{for } h^T x > 0 \\ A_2(x - x_{e2}) & \text{for } h^T x < 0 \end{cases}, \tag{1}$$

where $x, h \in \Re^2$, system matrices $A_1, A_2 \in \Re^{2\times 2}$, and $x_{e1}, x_{e2} \in \Re^2$ are such that $h^T x_{e1} > 0$ and $h^T x_{e2} < 0$. Let $L := \{x | h^T x = 0\}$ denote a hyperplane that partitions $\Re^2$ into two half-planes: the half-plane $P_1 := \{x|h^T x > 0\}$, and the half-plane $P_2 := \{x|h^T x < 0\}$. Then system (1) can be viewed as a variable-structure linear system with two linear systems separated by a hyperplane L and each with an equilibrium point, x_{e1} and x_{e2}.

Systems of the form (1) arise from many fields such as mechanics [2], electrical engineering [4], and automatic control [1, 3]. Such systems have drawn much attention recently due to their easy formulation and implementation. In contrast to seemingly simple appearance, the behaviors of piecewise linear systems are quite complicated and need further work to understand them.

In this article, we examine some 2-D examples of three classes of system (1) which possess limit cycle solutions and look into the mechanism for producing these limit cycles. The three classes considered are *unstable-focus-to-unstable-focus* (i.e., both the linear subsystems are of unstable focus type), *stable-focus-to-unstable-focus*, and *unstable-focus-to-saddle*. In order to achieve the goal, computer simulations are used to investigate some interesting bifurcation phenomena of the examples when some parameter in A_i or x_{ei} is varied.

Since system (1) is in general discontinuous, sliding motions are likely to exist. Indeed, we found that the sliding mode plays a crucial role in system behaviors. A special kind of limit cycle called *sliding limit cycle* can be produced due to the sliding mode. This kind of limit cycle has a portion on the sliding mode. One interesting property of it is that it can be reached in finite time by passing through the sliding mode.

Consider, for example, an example of unstable-focus-to-unstable-focus class:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & \mu \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, x_{e1} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}, x_{e2} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, h = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

where $\mu > 0$ is a parameter relating to the expansion rate of the first linear subsystem. As μ is varied from 0.5 to 0.9, the system experiences a homoclinic bifurcation followed by a saddle-node bifurcation of limit cycles and another homoclinic bifurcation, as shown in the figure. In the figure, "x" is an unatble equilibrium point on the sliding mode, and dashdot lines represent its stable manifold. Dashed lines stand for unstable limit cycles. Other trajectories are shown by solid lines. From the figure, on can see that before bifurcations, the system possesses sliding limit cycles.

From other examples, we found a gobal bifurcation induced by a local saddle-node bifurcation, and a different kind of saddle-node bifurcation between limit cycles which results from the touching rather than the coalesce of limit cycles. Detail explanations for these bifurcation phenomena will be provided in the full-length paper.

This work will reveal the complexity of the global behavior of system (1). Furthermore, it also provides the basis for further study of the piecewise linear systems in $\Re^3$, where we expect chaotic motions to occur.

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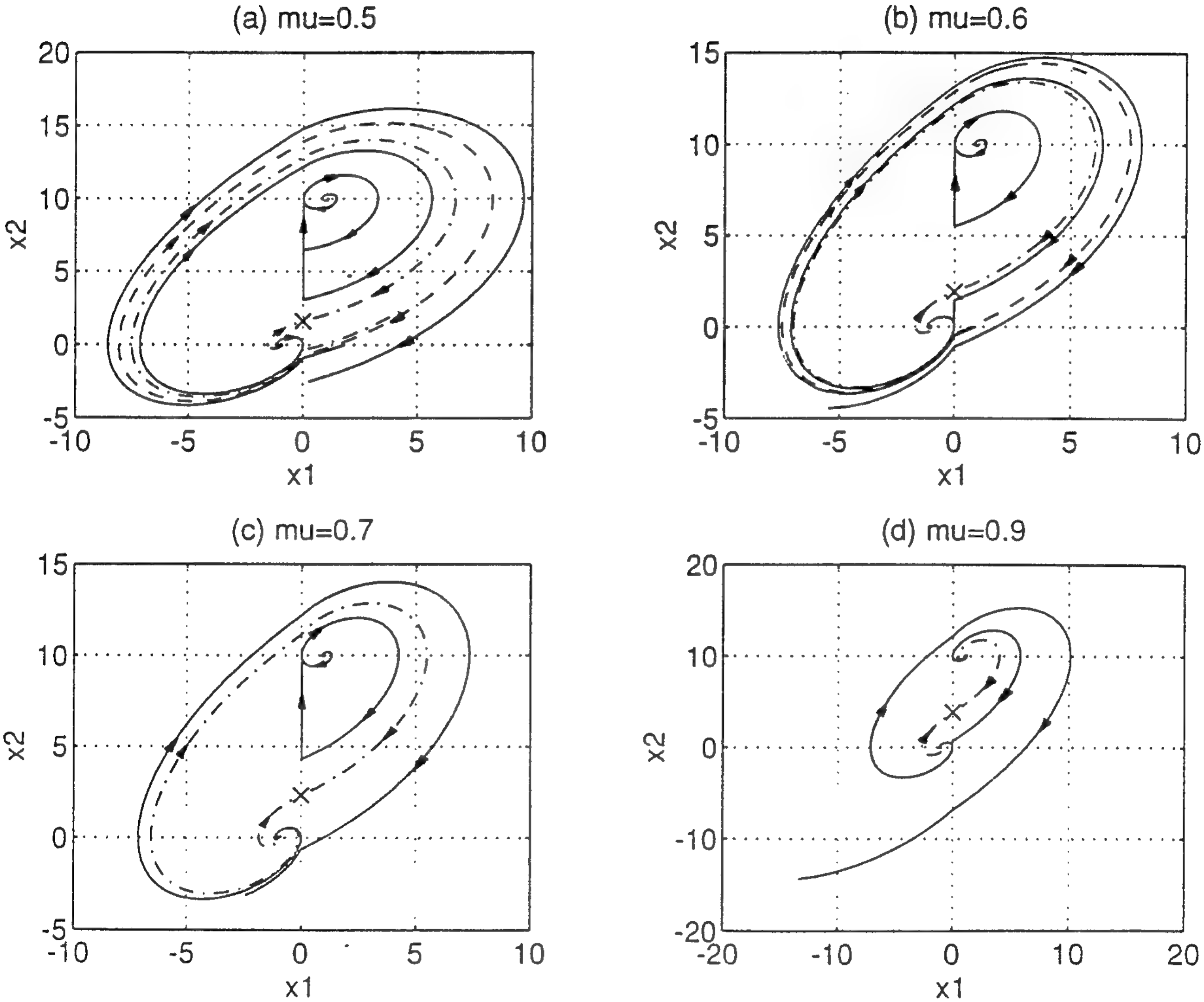
(a) mu=0.5
(b) mu=0.6
(c) mu=0.7
(d) mu=0.9
x1
x2

Jumps to resonance with a probabilistic outcome in systems subjected to deterministic excitation

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Abstract

Many nonlinear systems can experience jumps to resonance. Such jumps not only result in a qualitative change, but may often induce a substantial quantitative change in the response. In many systems jumps to resonance are purely deterministic in that there is always restabilization onto the resonance branch of the response curve. In this paper we show that jumps to resonance can be *indeterminate* where the outcome is essentially probabilistic, depending on upon the loading process. We show that such *indeterminate bifurcations* are a common phenomena in nonlinear dynamical systems. The basin and manifold organization prior to indeterminate bifurcations, as well as an analysis regarding the probabilitistic aspects of restabilzation are also presented. We show for example that *indeterminate saddle-node bifurcations* can arise when the unstable manifold of the saddle-node is heteroclinically tangled with the stable manifold of a distant saddle which itself is homoclinically tangled so it forms a fractal basin boundary between two remote attractors. At the bifurcation, a slowly evolving system will find itself sitting on a fractal basin boundary, and in the presence of infinitemesial noise the outcome is unpredictable in the sense that we cannot predict on to which co-existing attractor the system will settle upon. For parametrically excited systems, indeterminate bifurcations can occur when trivial solution is located on a fractal boundary when it loses its stability, say at a *sub-critical bifurcation*.

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Routes to escape from a potential energy well including experiments

by

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Abstract

Dynamical systems characterized by the possibility of escape from a local potential energy well occur in a great many physical problems including a rigid-arm pendulum passing over its inverted equilibrium position, snap-through buckling in arch and shell structures, and capsizing of ships. This is a thoroughly nonlinear problem and has received much recent attention especially using techniques based on numerical simulation. The current paper confirms many of these earlier findings using a mechanical experiment which mimics the behavior of a typical 'escape' nonlinear ordinary differential equation.

INTRODUCTION

The escape of trajectories from a local minimum of an underlying potential energy function is essentially a transient phenomenon. Given a single-degree-of-freedom system at rest in a position of stable equilibrium what excitation would cause the subsequent motion to overcome an adjacent barrier which defines the limit of the catchment region surrounding the minimum?

Escape occurs as the motion within the well grows 'large enough.' This is clearly more likely to occur when the forcing is 'large' in relation to some

although the governing equations may be well-defined, analytical solutions are limited, especially for nonlinear systems.

Previous research in this area began with the study of critical speeds in rotating systems (Lewis, 1932) and more recent work on linear dynamical systems includes transient testing using frequency sweeping (White, 1971), and resonant turbine blade behavior (Irretier and Leul, 1991). The effect of non-stationary influences on nonlinear systems includes predicting instabilities using transient dynamic effects (Virgin, 1986), nonlinear resonant effects in rotating shafts (Ishida *et al.*, 1987), approximate analytical results based on the perturbation method (Raman *et al.*, 1993), and chaotic behavior (Moslehy and Evan-Ivanowski, 1991).

The experimental system used to illustrate this type of behavior has been used successfully to illustrate a variety of nonlinear behavior (Gottwald *et al.*, 1992). Specifically the influence of frequency sweep rate on the resonant characteristics of the peak amplitude response is investigated and a comparison is made between experimental results and numerical simulation.

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Modeling and Bifurcations in Power System Dynamics

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In this paper we will demonstrate the importance of modeling in the appearance of complicated behaviour in power system dynamics. The appearance of the phenomena peculiar to nonlinear models such as bifurcations of different types depends critically on the degree of detail with which power system components are represented. We start with the classical representation of a single machine connected to an infinite busbar. The synchronous machine is represented by a constant voltage behind a transient reactance. As such the model is a second order differential equation with little information on the dynamics of the machine. If the busbar is assumed to have a quasi-infinite busbar, where its voltage is modulated in magnitude and phase, then the dynamics is enriched considerably. The complicated behaviour of the new system includes period doubling bifurcations, chaotic motions, and unbounded motions (loss of synchronism).

If the machine is still represented by a constant voltage behind transient reactance and connected to a load node which is also connected to an infinite busbar. If the load is assumed to have a component which is proportional to the derivative of the voltage magnitude or phase, then the three node system exhibits many local and global bifurcations. These bifurcations include saddle-node, Hopf, period-doubling leading to chaotic motions. For a region in the parameter space we have shown that a blue sky catastrophe ocuurs. The perodic solution disappears after colliding with a saddle point. This scenario shows the possibility of explaining the voltage collapse phenomenon as being a result of a global rather than a local bifurcation.

If the machine is represented by a dq model with one differential equation in addition to the swing equation, then an excitation system can be added. Such a model exhibits local bifurcations such as saddle node and Hopf bifurcations. Control action can be taken to change the dynamics of the system to remove the bifurcations from the region of interest in the parameter space.

Bifurcation and Chaos in the Duffing Oscillator with A PID Controller

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ABSTRACT

The problem of controlling chaos has attracted the attention of many investigators recently . Quite frequently chaos is a benefical feature as in some chemical or heat and mass transport problems , but in many other situations chaos is an undesirable phenomenon which may lead to vibrations , irregular operations etc. When we talk about controlling chaos , we mean to promote or eliminate it . There are mainly two methods for controlling chaos : the feedback method proposed by Ott , Grebogi , and York and nonfeedback method proposed by T. Kapitamik and other scholars . Duffing oscillator has been used to illustrate these two methods . So it is useful and interesting to investigate the dynamical behavior of the oscillator when subject to a PID controller , for PID controllers are still widely used in engineering . The Duffing oscillator can be described as

$$\ddot{y}+y+\mu\dot{y}+\delta y=u\ (t) \tag{1}$$

when $u\ (t)\ =f\cos\omega t$, it is a forced vibration problem . A H.Nayfeh once used perturbation method to investigate the complicated dynamical behavior when the parameters is small ($\mu,\ \delta,\ f\langle\langle 1$) . P. Holmes studied the bifurcation and chaos of a Duffing oscillator with a first order linear feedback controller . he showed that the system does not always has asymptotic stable solutions even no $u\ (t)$ appears , moreover , when subject to periodic position feedback chaotic responses are possible . P. A. Cook once pointed out that integral action feedback may lead chaos .

In our paper , we study the bifurcation and chaos of Duffing oscillator

with a PID controller . A PID controller can be written as

$$u(t) = k_p e(t) + k_d \frac{de(t)}{dt} + k_i \int_0^t e(t)\,dt \tag{2}$$

where $e(t) = y_r(t) - y(t)$, and $y_r(t)$ represents the reference input signal . k_p, k_d and k_i are the coefficients of proportion , differentiation and integration respectively . We further write the equations of system as

$$\begin{cases} \ddot{y} + by + a\dot{y} + \delta y^3 = -kv + r(t) \\ \dot{v} = y \end{cases} \tag{3}$$

where $b = 1 + k_p$, $a = \mu + k_d$, $k = k_i$, $v = \int_0^t y dt$, $r(t) = k_p y_r(t) + k_d y_r(t) + k_i \int_0^t y_r(t)\,dt$, we first set $r(t) \equiv 0$ and study the intrinstic "free" dynamics of the system . For convenience we rewrite (3) as a first order system .

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -bx_1 - ax_2 - kx_3 - \delta x_1^3 \\ \dot{x}_3 = x_1 \end{cases} \tag{4}$$

where $x_1 = y$, $x_2 = dy/dt$ and $x_3 = v$,

We find that the only fixed point is X_b (0, 0, 0) , when $ab = k$, Hopf bifurcation occurs. Using center manifold theorem and other techniques , we conclude that :

(I) .when the Duffing oscillator has hard spring nonlinearity term ($\delta > 0$), if $ab < k$ and $|ab - k|$ small enough , the system (4) has a stable limit cycle near the fixed point X_b (0, 0, 0)

(II) . when the duffing oscillator has a soft spring nonlinearity term ($\delta < 0$), if $ab > k$ and $|ab - k|$ small enough, there is a unstable limit cycle near the fixed point X_b (0, 0, 0) .The fixed point is local stable but not global stable .

(III) . no chaos occurs in the practical ranges ($k_p > 0$, $k_d > 0$, $k_i > 0$) .

When $r(t)$ is a T-periodic step function chaotic response can be obtained . The numerical simulation is done and it supports the theoretical conclusion . We further use Wavelet Transform to analyze the chaotic response .

Tuesday, June 14

1330-1510

Session 9. Chaos

ON THE PERIOD-DOUBLING BIFURCATIONS IN THE DUFFING'S OSCILLATOR WITH NEGATIVE LINEAR STIFFNESS

by

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ABSTRACT

The phenomenon of period-doubling bifurcations in the Duffing's oscillator with negative linear stiffness is investigated with the aid of approximate analytical methods and computer simulation. Making use of a Hill's type variational equation together with the ideas drawn out from Floquet theory, it is found that a particular type of subharmonic instability is the one that is responsible for the occurrence of period-doublings in this system.

In this paper, we investigate, using only the first-harmonic approximate solution, the steady-state behavior of a double-well potential oscillator focusing attention on its local oscillations and their stability. The governing equation for the considered system is :

$$\ddot{x} + 2\mu\dot{x} - \frac{1}{2}(x - x^3) = F\cos(\omega t+\beta)$$

Based on results obtained from Floquet theory, it is shown that the stability analysis of the steady-state solutions via a Hill's type variational equation, answers to a large extent the question of the existence of the period-doubling bifurcations observed numerically (in the frequency range $1.0 \geq \omega \geq 0.95$).

The following criterion for the P.D.B's is developed. This criterion relates the system parameters necessary for period-doubling:

$$F^2 \geq 4\mu^2\omega^2 C_1^2 + C_1^2(1-\omega^2-\frac{15}{8}C_1^2)^2$$

where

$$C_1^2 = \frac{1}{15}\left\{7-\omega^2-\left[9+(6-40\mu^2)\omega^2-\frac{9}{2}\omega^4\right]^{1/2}\right\}$$

There is no guarantee that these P.D.B's will develop into chaos. In those cases where chaos is preceded by P.D.B's, this criterion could predict a lower threshold for chaos. This is confirmed by the striking similar nature of the true (numerical) chaotic threshold and the present criterion. Also there is good agreement between Moon's criterion for chaos and the present one. Based on this, it is suggested that the former criterion predicts period-doubling rather than chaos.

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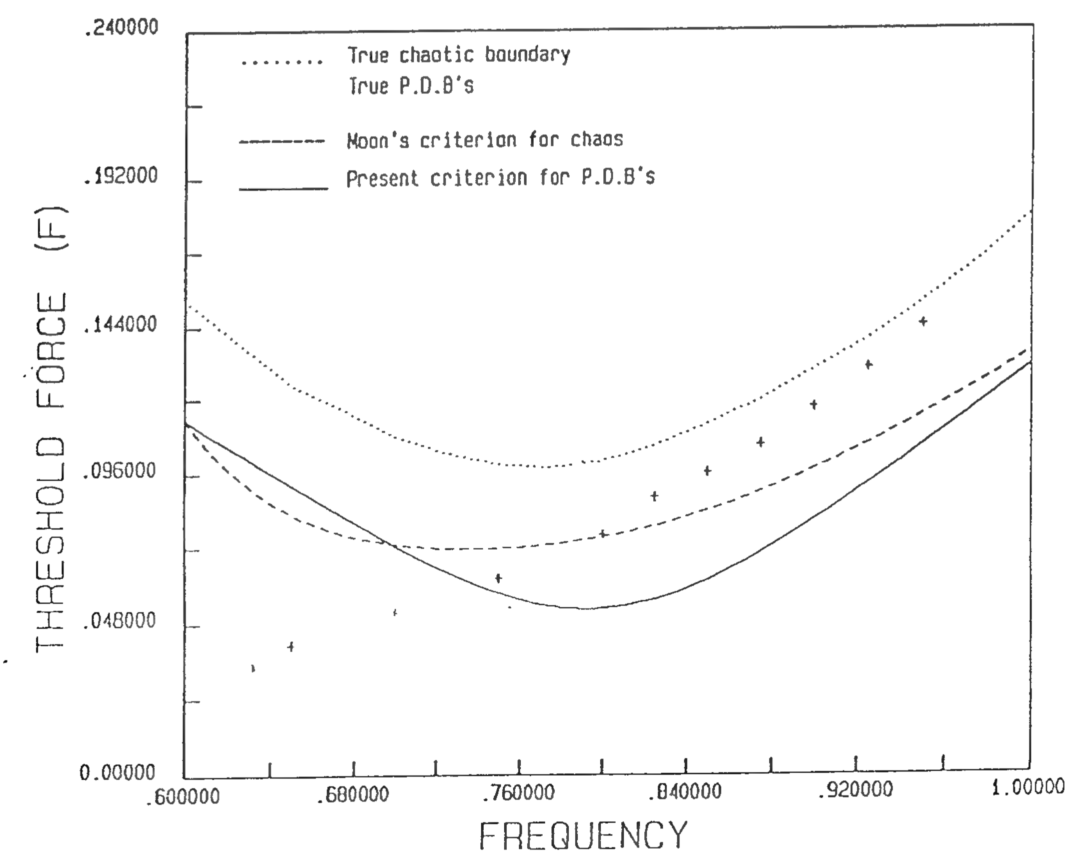

Chaotic Unpredictability of Elastic-Plastic Response to Impact Loading

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Abstract

Computing permanent deflections of a beam or plate due to transverse impact or other type of short pulse loading, such that plastic strains occur, ordinarily presents no great difficulty. Robust finite element/difference programs are available for nonlinear geometric and material behavior. Simple approximation techniques are useful when plastic deformations predominate. The present paper shows some recent results in a class of problems for which no meaningful solution exists, numerical or analytical, for certain ranges of the parameters: the permanent deflection *cannot be predicted*. No refinement in computational technique will lead to a meaningful result.

The meaning of "unpredictability" is illustrated in Fig. 2 [1]. This shows displacements computed for the simple two-degree-of-freedom beam model sketched in Fig. 1, for two pulses of force at the midpoint, each with duration 0.5 msec. In one case the force during the pulse is 2500.000001 N, in the other case it is 2500.000002 N. In the first case the final deflection is positive. The second pulse, differing from the first in the tenth significant figure, predicts a negative outcome. This is not a bifurcation or otherwise "special" load value. The same abnormal sensitivity is found at pulse forces in a wide range, say between 1600 N and 3400 N, for this particular model. A minute change of the pulse strength parameter may (or may not) cause the predicted final deflection to change its sign and magnitude.

The significance to analysts and designers of an extreme sensitivity to parameters, and the implied unpredictability, is obvious. Of course, unpredictability is hardly unknown in mechanical systems. We live in an intrinsically nonlinear world. In celestial mechanics, predictions are valid only for finite times. The present problem involves a more conventional engineering task: to estimate permanent deformations due to transverse impact or short pulse loading on a beam or plate. To find that seemingly minor changes in conditions can transform a readily solvable problem into one that is inherently unsolvable, is unexpected. Apart from some early Russian work [2], it seems not to have been noticed prior to our accidental observation in 1984 [3]. The primary requirement is that the supports provide constraint against axial motions, with consequent geometrical nonlinearity. In addition, plastic extensional strains must be induced in a certain range of small magnitudes, the pulse must be sufficiently short, damping must be sufficiently weak, and the calculation must employ a multi-degree of freedom model of the structure [4].

The source of the unpredictability is the *chaotic* nature of the response during a pre-transitional phase [5]. The end fixity means that finite transverse displacements require extensional strains at the middle surface. For our 2dof model, the equations of motion are two coupled nonlinear equations. The plastic strains change during an initial "shakedown" period. After they become constant, the equations are of standard Duffing type. The small plastic strains convert the original beam to a shallow arch. With two or more degrees of freedom, it can exhibit chaotic vibrations. Assuming (light) damping, there is a transition (illustrated in Fig. 2) to a motion that remains either positive or negative, becoming more regular as it approaches the final rest state.

The paper illustrates that unpredictability in our problem corresponds to the *independence of scale* of a diagram that shows the incidence of positive and negative final displacements when the pulse force is varied in small steps over an interval. A first scale gives these quantities over a range of 200 N from 2500 N to 2700 N, divided into 100 steps of 2 N each. Expansions of the scale of 1.0E2, 1.0E4, and 1.0E6 are used; thus the fourth scale shows results in the interval from

2500.0000 to 2500.0001 N at 100 steps of 0.000001 N each. All four figures are remarkably similar. The occurrence of negative outcomes can be expressed numerically by computing Mandelbrot's "self-similarity" fractal dimension [6]. This turns out to be close to 0.78 for all scales [1]. This may be compared with the dimension of a line composed of "Cantor dust" particles, namely log2/log3, or about 0.63, for all scales. The same quantity for a single degree of freedom model is unity; there is no scale independence.

In this problem, the simplest general approach to understanding a wide range of chaotic and conditionally periodic behaviors is by energy methods [7]. The paper will briefly review these and other aspects of the response behavior both of the damped model, and that in which the damping is taken as zero. The latter allows standard tests of chaotic behavior (e.g. Lyapunov exponents) that have meaning also for the chaotic stage of the lightly damped structure. It allows construction of "surface-of-section" plots for fixed total energies of the Hamiltonian system defined by the equations of motion for fixed plastic strains and zero damping [8, 9]. For this system sensitive and highly complex dependence on initial conditions is exhibited in the patterns derived from intersection points of trajectories with particular planes in phase space. Since the model of Figure 1 is somewhat artificial, it is of interest to note the similar equations derived by Galerkin's method, which furnish qualitatively similar results [10].

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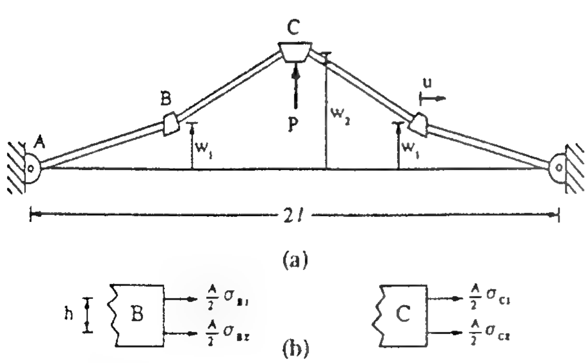

Figure 1 Two-degree-of-freedom beam model. (a) symmetric deflected configuration; (b) deformable cells at quarterpoints B and midpoint C, treated as sandwich beam sections.

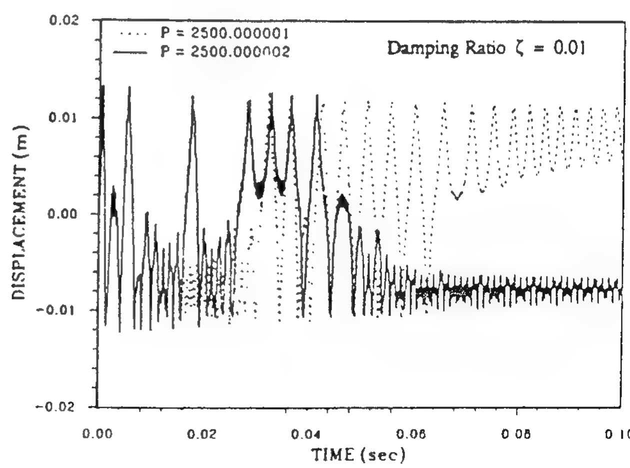

Figure 2 Examples of time histories of midpoint displacement: two cases.

GLOBAL BIFURCATIONS AND CHAOS IN THE RESONANT RESPONSE OF A STRUCTURE WITH CYCLIC SYMMETRY

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Periodic structures with cyclic symmetry are often used as idealized models of physical systems and one such model structure is studied in this work. It consists of n identical particles, arranged in a ring, interconnected by extensional springs with nonlinear stiffness characteristics, and hinged to the ground individually by nonlinear torsional springs. These cyclic structures that, in their linear approximations, are known to possess pairwise double degenerate natural frequencies with orthogonal normal modes, are studied for their forced response when nonlinearities are taken into account.

The method of averaging is used to study nonlinear interactions between the pairs of modes with identical natural frequencies. The external harmonic excitation is spatially distributed like one of the two modes and is orthogonal to the other mode. A careful bifurcation analysis of the amplitude equations is undertaken in the case of resonant forcing. The response of the structure is dependent on the amplitude of forcing, the excitation frequency, and the damping present. For sufficiently large forcing, the response does not remain restricted to the directly excited mode, as both the directly excited and the orthogonal modes participate in it. These coupled-mode responses arise due to pitchfork bifurcations from the single-mode responses and represent traveling wave solutions for the structure. Depending on the amount of damping, the coupled-mode responses can undergo Hopf bifurcations leading to complicated amplitude-modulated motions of the structure. The amplitude-modulated motions exhibit period-doubling bifurcations to chaotic amplitude-modulations, multiple chaotic attractors as well as "crisis". The existence of chaotic amplitude dynamics is related to the presence of Sil'nikov-type conditions for the averaged equations. The effect of weak linear mistunings on the structure of solutions is also investigated.

Global bifurcations in the averaged equations for the weakly cyclic structure are studied using a generalization of the Melnikov method. Homoclinic orbits are shown to break, generating Smale horseshoes, resulting in chaotic phenomena. We also use a new Global perturbation technique developed by Kovacic and Wiggins, that is a combination of higher dimensional Melnikov method and geometrical singular perturbations, for detecting the parameter values for which a Sil'nikov-type homoclinic orbit exists. With appropriate conditions on eigenvalues, this implies Sil'nikov-type chaos for the averaged equations.

NONLINEAR AND CHAOTIC DYNAMICS

OF ARTICULATED CYLINDERS IN CONFINED AXIAL FLOW

R. M. Botez and M.P. Païdoussis

Flexible or articulated cylindrical components in a flow–cooled cylindrical conduit are commonly found in many engineering systems and often experience fluidelastic oscillations and failure (Païdoussis 1980). Impacting with the outer conduit is often involved and hence strongly nonlinear forces. It is therefore reasonable to suspect the existence of interesting nonlinear behaviour and chaos in such systems.

This study deals with the planar dynamics of a vertical articulated system of two or three cylinders interconnected by rotational springs and with a free downstream end, centrally located in a pipe conveying fluid downward in the narrow annular space in–between. Several analytical models have been developed: (a) a linearized model in which the only nonlinearity is due to impacting with the outer pipe; (b) a nonlinear model in which geometric and fluid–dynamic nonlinearities are approximately accounted for; (c) variants of (a) and (b) accordingly as to whether impacting is modelled by a trilinear spring, a cubic spring approximation thereof, or a restitution coefficient model. The system was studied by varying mainly the dimensionless flow velocity, u, the number of articulations, N, the dimensionless annular gap, h, and the free–end shape parameter f. In several ranges of parameters there is reasonable qualitative (and in some cases quantitative) similarity in the dynamics according to the above–mentioned models.

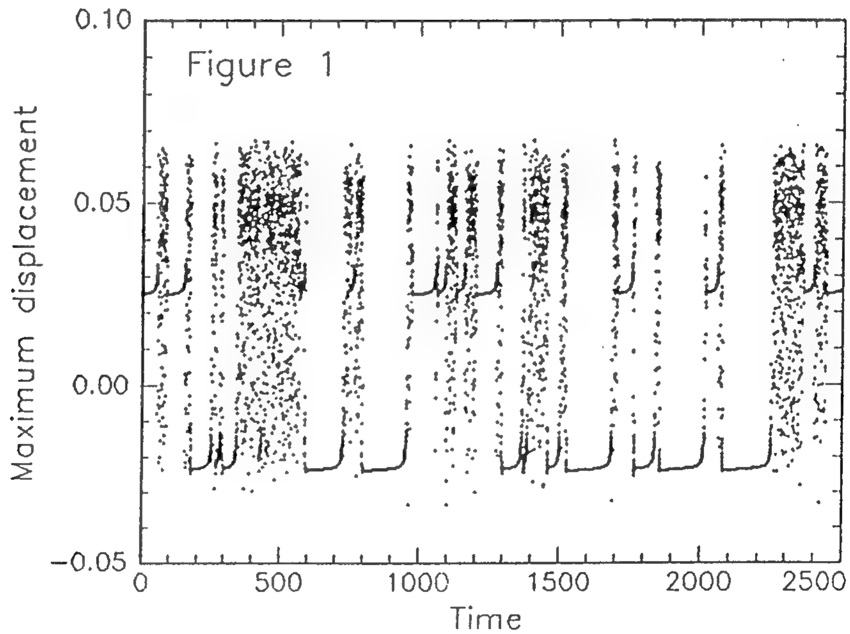
Depending on the system parameters and u, periodic, quasi–periodic and chaotic oscillations were found to exist (Païdoussis and Botez 1993), confirmed by phase–plane portraits and bifurcation diagrams, time traces and power spectral densities, Poincaré maps and Lyapunov exponents. Three routes to chaos were found: the period–doubling cascade, the quasi–periodic route, and type III intermittency.

In this paper, three cases associated with different routes to chaos are presented as follows: in the first case ($N = 2$, $h = 0.5$, $f = 0.8$) the route to chaos is via a cascade of period–doubling bifurcations; in the second case ($N = 2$, $h = 0.2$, $f = 0$, which corresponds physically to a smaller annular space and to a blunt end), chaos is associated to periodic motions around two symmetric points; and in the third case ($N = 3$, $h = 0.5$, $f = 0.4$) chaos is associated with type III intermittency.

The maximum angular displacement of the first cylinder is presented as a function of time in Figure 1 for the intermittency case (case 3). In this figure one may see

the "turbulent fluctuations" in the oscillation, represented by the regions with many points, interrupted by regions of more regular oscillations or "laminar" phases. These laminar phases are in fact associated with slowly growing limit cycles. The statistical distribution of the lengths of laminar phases versus time interval as well as the first and second return maps are studied, leading to the conclusion that the intermittency in this case is of type III.

Apart of the use of centre–manifold theory in the case of $N = 2$, most of the foregoing was obtained numerically by Runge–Kutta integration of the equations of motion. More recently, some of these systems were also studied by means of the AUTO software (Doedel 1986) which, among other things, allows a more systematic construction of more complete bifurcation diagrams than our own software. Since in some of the systems studied the mechanism leading to chaos is still unclear, further studies with AUTO and other software are being pursued to hopefully obtain a better understanding.



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Chaotic Dynamics of Quadratic Systems with 1:2 Internal Resonances

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Two degrees-of-freedom dynamical systems often have quadratic terms in their equations of motion. The quadratic nonlinearities arise due to inertial effects of large motions, and due to the phenomena associated with centrifugal and coriolis forces [Sethna, 1965]. Interesting response of the systems is observed when the two modes of vibration of the system get coupled through the nonlinear terms. This situation arises when the linear natural frequencies of the system are in the ratios 1:2 or 2:1 (internal resonance). In these systems with internal resonances, complicated motions are observed when the frequency of external excitation is close to one of the linear natural frequencies of the system.

We study systems with subharmonic 1:2 internal resonance. The frequency of the first mode of vibration is taken close to 1, the normalized frequency of excitation, and that of the second mode, close to $\frac{1}{2}$. The method of averaging [Murdock, 1991; Wiggins, 1990] is used to reduce the nonautonomous system to an autonomous system. The autonomous system captures the essential dynamics of the original system for sufficiently small motions near resonance. Fixed points of the averaged system correspond to periodic solutions of the original system and periodic solutions of the averaged system imply amplitude-modulated motions for the original system. Chaotic solutions of the averaged equations imply chaotic amplitude-modulated responses for the original system.

For the undamped averaged system, we use an extension of Melnikov's method, developed for autonomous Hamiltonian systems by Holmes and Marsden [1982], and presented in considerable detail in Wiggins [1988], to analytically predict the parameter range for which chaos exists in the system. The underlying theme of Melnikov's idea is to consider an unperturbed Hamiltonian system, having a hyperbolic fixed point connected to itself by

a homoclinic orbit. On perturbing this system with time periodic perturbation (not necessarily Hamiltonian), the hyperbolic fixed point becomes a hyperbolic periodic orbit, whose stable and unstable manifolds may intersect transversely, giving rise to Smale horseshoes and hence complex invariant sets. The distance between the stable and the unstable manifolds can be calculated using the Melnikov function or Melnikov Integral. The parameter values for which the Melnikov function has simple zeroes, gives the desired parameter regions. This analysis follows closely the work of Tien, Namachchivaya and Bajaj [1993].

It is found that any small external excitation breaks the heteroclinic orbits and leads to transverse intersections of the stable and unstable manifolds, leading to chaotic dynamics of the averaged system. These results are verified by numerical simulations of the averaged equations, and it is seen that the chaotic responses persist even in the presence of sufficiently small damping (as compared to the amplitude of the external forcing), although the analysis is not valid for the damped case.

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Tuesday, June 14

1530-1710

Session 10. Multibody Dynamics

EXPERIMENTAL STUDY OF A COMPLEX NONLINEAR MECHANICAL SYSTEM

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ABSTRACT

Classes of nontrivial nonlinear vibratory systems occur frequently in practice. Analyses are generally (over)simplifications and hence there is interest in substantive experimental analysis.

An experimental analysis was carried out on the response of a multi-degree-of-freedom nonlinear mechanical system. The aim of the analysis was to conduct a study of the system's behaviour in the time, frequency and phase space domains. The system was designed and built by coupling an array of 6 planar rigid pendulums by nonlinear springs. Each pendulum was supported by a thin plane spring, which restricted the oscillator to a plane motion and also offered space to attach strain gauges. The coupling springs, the characteristics of which could be changed by varying their lengths and thicknesses were made of high strength spring steel shims arranged to operate in a post buckled state, resulting in their nonlinear characteristic.

Analysis of the data was carried out in both the time and frequency domains. The system was studied in free response, and also when subjected to harmonic and random excitations. Time domain techniques included reconstructed phase space (and measures on it) as well as delay maps. The spectral approach included higher order spectra for harmonic as well as for random excitation.

When studying the free response a progressively more complex system was analysed starting with one, followed by two and finally with all 6 oscillators, coupled with nonlinear springs. The subharmonic response as well as combinational frequencies were resolved.

Applying deterministic harmonic excitation at twice the first natural frequency of the system created a saturation phenomenon which resulted in an extremely high response at half of the excitation frequency. These transitions are illustrated for different sets of the coupling springs in the time - frequency domain using short time spectrograms. This is achieved by varying an input parameter, such as the amplitude of the fixed forcing frequency, with time. This allows a clear indication to be obtained of the changes that occur when the input is varied.

* This research was carried out when the author was on leave at ISVR, University of Southampton

When applying harmonic excitation at another single excitation frequency at a high forcing level the system responded with a spectrum containing many frequencies. This is shown in the time and frequency domains and especially using higher order spectra (specifically bispectra and bicoherence). As many combinational frequencies were present ordinary power spectra could extract them well but the bispectra - bicoherence showed additional information about quadratic coupling of two separate frequencies. Bicoherences were also calculated in the case of the random excitation, where the rise in the excitation level resulted in stronger mode coupling and this also resulted in higher magnitude value of the bicoherence.

When applying deterministic and random excitation to the built multi-degree-of-freedom nonlinear mechanical system measures from chaotic dynamics theory including the integral of correlation dimension were applied. Long duration time histories were acquired and the abstract multidimensional phase space reconstructed using delay-time embedding techniques. Different delay-times were taken into account and integrals of correlation dimension calculated for different embedding dimensions.

Convergence was found in correlation dimension estimates with increasing embedding dimension in the case of deterministic excitation. No such convergence was found when applying random excitation. Increasing the forcing amplitude for deterministic excitation resulted in the responses becoming more complex. This increasing complexity can be seen from time histories alone, power spectra, bicoherences, delay-time plots and increasing slopes in the integrals of correlation dimension.

The increase in the estimates of the correlation dimension were found for two different sets of coupling springs with the increase of excitation amplitudes at deterministic forcing. Although extracting slopes from the integrals of correlation dimension requires (theoretically) an infinite number of reconstructed attractor points and noise free data, reliable estimates can be estimated for low dimensional dynamics.

By acquiring more output signals (simultaneously) it was possible to apply the delay-time reconstruction technique to many corresponding time histories and compare the estimated values of correlation dimensions. Doing so for three signals at three different locations in the system confirmed the invariant nature of the underlying attractor in the case of random excitation.

A NONLINEAR FINITE ELEMENT APPROACH FOR KINETO-STATIC ANALYSIS OF MULTIBODY SYSTEMS

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ABSTRACT

Methods that treat rigid/flexible multibody systems undergoing large motion as well as deformations usually utilize a mixed set of rigid body coordinates and flexible coordinates. The numerical solution of the corresponding governing equations of motion is often accompanied with inefficiencies and instabilities due to the large number of state variables, differences in the magnitudes of the rigid and flexible body coordinates, and the time dependencies of the mass and stiffness matrices. The kineto-static methodology of this paper treats a multibody mechanical system to consist of two collections of bodies. One collection contains bulky compact solids that can be modelled as rigid bodies, while the second collection contains relatively flexible bodies that are deformable. The equations of motion are formulated for the rigid bodies only, whose solution predicts the gross motion of the rigid bodies and also the displacement of the nodes at the boundaries between the rigid and the flexible bodies. A nonlinear finite element model of the deformable bodies or the structural part of the system is constructed. A combined incremental and iterative process is used at each time step of the numerical integration process for calculating out-of-balance forces, updating the tangent stiffness matrix, and calculating incremental displacements, stresses, and strains. A mixed boundary condition finite element problem is then formulated at each time step whose known quantities are the displacement of the nodes at the boundary and the internal loads acting on the structure, and its unknowns are the deformed shape of the entire structure and the loads (forces and moments) at the boundary of the rigid and flexible bodies. Partitioning techniques are used to solve the linear systems of equations for the unknowns. The loads at the boundary are then treated as external loads acting on the rigid multibody system, and the numerical solution of the rigid multibody system governing equations of motion is continued. A general-purpose nonlinear finite element analysis code that performs the kineto-static analysis has been developed.

The methodology discussed earlier is very much suitable in modelling and predicting the crash responses of multibody systems since both nonlinear and large gross motion and deformations are encountered. Therefore, it has been adopted for the studies of the dynamic responses of ground vehicle or aircraft occupants in different crash scenarios. Occupant models are robust tools for gaining insight into the gross motions and for evaluating the loads and deformations of their critical parts in studies of crashworthiness. The knowledge of occupant responses will help in understanding and determining the type and probable causes of injuries that may be sustained. An important issue in occupant modeling is how the large motion of rigid segments of occupants such as the limbs and the small deformations of flexible bodies such as the spine column are handled. One of the most dangerous modes of injury is the amount of compressive loads that is encountered by the spine. This situation is much more evident in aircraft crashes, compared to the automobile accidents, since the aircraft crashes have a dominant component of the vertical impact load. The kineto-static analysis methodology of multibody systems with flexible structures undergoing large motion and complicated structural deformations has then been used for this application. Rigid multibody dynamics is used to predict gross motion of the body parts including the pelvis and the thorax. The displacements at the boundaries of the these two parts with the spine are then evaluated at each time step. Nonlinear finite element analysis with mixed boundary conditions is then performed to determine the corresponding loads and deformations of the spine.

Based on the developed method, a mathematical model of the occupant with a nonlinear finite element model of the lumbar spine is developed for a Hybrid II (Part 572) anthropomorphic test dummy. The lumbar spine model is then incorporated into a gross motion occupant model. The analytical results are correlated with the experimental results from the impact sled tests. Comparison of the results has shown closer match of the analyses to the experiments. With the validated occupant model containing the lumbar spine, the gross motion of occupant segments, including displacements, velocities and accelerations are evaluated. The spinal axial loads, bending moments, shear forces, internal forces, nodal forces, and deformation time histories are also determined. The gross motion of occupant segments, including displacements, velocities, and accelerations may also be evaluated. This detailed information helps in assessing the level of spinal injury, determining mechanisms of spinal injury, and designing better occupant safety devices.

A SYMBOLIC-NUMERICAL APPROACH TO CHARACTERIZE THE STABILITY AND CONTROL THE DYNAMICS OF A FOUR-WHEEL-STEERING VEHICLE

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A symbolic-numerical approach is used to analyze the response of a road vehicle equipped with four-wheel-steering system using a nonlinear model with a low number of DOF including yaw, sideslip, and roll. A nonlinear control law for the rear wheel is derived in closed-form, and it shows better characteristics than its linear equivalent under steering and braking maneuvers. The stability of the straight line motion for four-wheel steering and front-wheel steering vehicles is analyzed using symbolic manipulation, to characterize the stability in terms of eigenvalues computed in closed-form. This approach can have significant applications to enhance the design process by providing formulas to asses dynamic stability.

The ever growing demand for road vehicles with better performance has brought the traditional front wheel steering system under scrutiny. One result of that scrutiny that has gained popularity is the four-wheel steering system (4WS). Furukawa et al. (1989) presented a complete review of the development of the 4WS for automobiles, and a few auto makers have been manufacturing 4WS vehicles for a number of years.

Four-Wheel steering systems thus have the potential to become an essential vehicle design technology. However, a number of theoretical and practical concerns remain to be resolved before the technology becomes widespread. A point of controversy among researchers has been the way the rear wheels should be steered in order to optimize the handling performance and stability of the vehicle. Whitehead (1988), Bernard (1988), Nalecz et al. (1988), and Sridhar et al. (1992) among others have proposed various

control schemes. Two main characteristics tend to distinguish the approaches to the analysis of this problem: the control law to be used to steer the rear wheels, and the model used to assess the control law's impact on the vehicle's dynamics.

The present work considers a fully nonlinear model of the response of a road vehicle equipped with 4WS. The vehicle is modeled with a low number of degrees-of-freedom, including yaw, sideslip, and roll. A symbolic approach is used to allow the implementation of analytical and numerical techniques, taking advantage of the availability of symbolic manipulation software (MAPLE, Char et al. 1991). This work expands an initial nonlinear model (Sanchez, 93) which was developed for low levels of acceleration and did not include the roll mode. Closed-form results can be very valuable for design applications, providing designers with formulas that can be used to assess dynamic stability. At the same time, it is shown that the symbolic model can be used to perform numerical simulations to evaluate the impact of the control law. Thus, the symbolic-numerical approach provides an environment that can significantly enhance engineering analysis and design.

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SYMBOLIC MODELLING OF FLEXIBLE ROBOTS AND IDENTIFICATION OF DYNAMIC PARAMETERS

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The search for a minimal ratio "Robot weight/Payload weight" in order to improve the dynamic performance of the robots has led to take into account the bodies' flexibilities in the modelling. The aim of this paper is to present a methodology to obtain, in a formal way, the energy and dynamic models of an open loop robot structure with flexible bodies.

The interest of the use of symbolic computation for the rigid robots structures modelling has already been proved. A computer aided approach leads to a gain in time on the calculus of final model by eliminating the operations which lead to a null result and by allowing intermediary variables [1]. The model obtained can be used immediately in control algorithm where the model is computed at each step. Moreover formal identification models can be obtained easily.

The modelling of the structural flexibilities of manipulators has the same purposes. Moreover, when the number of flexible bodies is greater than three, the manual development of models is tedious and prone to errors. The advantages of a computer aided-approach are that it : minimises human burden, avoids errors, allows to manipulate formulas easily, can deal with more terms (higher order and elastic modes) and one can keep the number and the type of modes in a symbolic form in order to test different modes combinations in simulation.[2-4]

In this article, we present in a first part the method used to model flexible robots. The formulation is based on Lagrangian co-ordinates. The parameters of the model are the joint variables and the elastic degrees of freedom of the bodies. The formulation proposed is independent from the choice of discretization type (assumed modes or finite elements methods). The elastic bodies are treated as Euler-Bernoulli beams and their motions are referred to the position of the undeformed corresponding link [5-6]. Finally the dynamic equations of the robot are obtained using the Lagrange equations. We show that the models we obtain are linear with respect to a standard set of parameters which are the inertial and stiffness parameters of the structure. In reference 7, we

have developed a formal approach in order to determine the standard parameters' sufficient conditions of minimality. The minimality of the standard set of parameters ensures the robustness of the identification process. [8]

The generation of formal models of flexible robots in a form suitable for identification or control is then treated. A particular attention is given to the development order of the model in relation to the elastic degrees of freedom. We focus our study on the development order of the intermediary calculations in order to obtain a final model which is coherent in terms of development order. Two models are discussed : an energy model used for the standard parameters' identification and a dynamic model used either for the identification or for the control of the robot. In the case of the dynamic model, we study the effects of a model simplification in terms of development order on the properties of the definite positive mass matrix of the structure.

Next, we present how we manage to implement the calculus of the models in a form suitable for symbolic computation. The discretization is based on the finite elements method. Each body is modelled by a two nodes beam finite element. The models' final expressions are calculated in function of elementary matrices coefficients (intermediate terms).

Lastly, we present an application of our method. We determine the energy and dynamic models of a three joint robot (prismatic, revolute, revolute) : the development at order two of the energy model leads to 601 operations and the dynamic model written at order one contains 1437 operations. The small number of operations makes it possible to calculate the dynamic model in real time, and allows therefore the implementation of a control law based on non-linear decoupling theory as described in reference 6.

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Experimental and Numerical Investigation of the Pitch and Bounce Response of a Railroad Vehicle

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Abstract. In this paper a model of a freight car is constructed and simulated over a test track, which was designed to investigate vehicular pitch and bounce response. The two dimensional model consisted of three rigid bodies, representing the car body and the two trucks, and four massless bodies, representing the track profile under each axle. The connections or suspensions between bodies consisted of non-linear stiffness and damping. A quasi static simulation was performed, in the sense that forward motion was ignored, since the vehicle was assumed to be travelling at a constant speed. This simulation attempted to duplicate an actual test track, in which ten sinusoidal vertical profile humps, with 39 foot wavelength and 0.75 inches amplitude, were constructed. An actual test of a 70-ton freight car over this test track was performed at various speeds. The car was instrumented to measure the response to vertical track profile irregularities. The model was simulated at the same speeds and a comparison was made between the response of the vehicle and the model simulation. The response of the vehicle, as measured with an actual 70-ton freight car and the model simulation, compared quit favorably throughout the speed range at which they were examined.

Wednesday, June 15

0830-1010

Session 11. Analytical Methods I

CONTROL OF DYNAMICAL SYSTEMS SUBJECTED TO PERIODIC PARAMETRIC EXCITATIONS

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Abstract: Some techniques in the design of controllers for linear as well as nonlinear dynamic systems subjected to periodic parametric excitations is presented. The control of linear periodic systems is quite challenging due to its time-varying nature. In the past, several methodologies for control system designs of time-varying systems have been reported[1- 5]. Invariably, these methods are based on transforming the original system into a suitable canonical form so that some of the special properties of the canonical system can be utilized for controller designs. However, such transformations, if they exist, are not unique and are tedious to implement, especially for higher dimensional systems. In this paper, the idea is to utilize the well-known Liapunov-Floquet (L-F) transformation such that the original time-varying linear control problem can be converted into a form which can be studied via time-invariant methods of control theory. When the L-F transformation is applied to a quasilinear periodic system, a dynamically similar system is obtained whose linear part is time-invariant and the nonlinear part consists of coefficients which are periodic. First, a procedure for computing the L-F transformation matrices[6] for general linear periodic systems is outlined. In this procedure the state transition matrices are expressed in terms of Chebyshev polynomials[7] which permits the computation of L-F transformation matrices as explicit functions of time. Secondly, it is shown that the controllers can be designed via full state or output feedback using principles of pole placement and/or optimal control theory in the transformed domain for linear systems. Once the control gains are obtained in the time-invariant form, the time-varying periodic gains of the original system is obtained by employing error minimization criteria between the original and transformed systems[8-9]. In the presence of structural perturbations, it is found that the above control design is robust and a measure of bounds for the structural perturbations can also be provided. In the context of performance improvements, linear control design alone may not meet the desired specifications of the nonlinear periodic systems due to the time-varying nature of the problem. Therefore, to improve the controlled response of the nonlinear system, a nonlinear time-varying controller is also designed and incorporated via the Liapunov's Direct Method. The performance of the linear and the dual (linear + nonlinear) controllers are compared. Noticeably, the combination of linear and nonlinear controllers based on L-F transformation approach has been found to have better performance and robustness characteristics. The benefits of this technique is demonstrated through two examples. The first example belongs to the class of commutative systems while the second one is a double inverted pendulum subjected to periodic loadings. It is found that the linear and nonlinear control strategies perform well in the presence of structural perturbations.

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Fig 1 : A Double Inverted Pendulum Subjected to a Periodic Follower Force

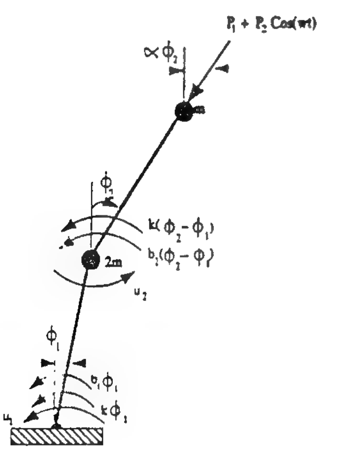


Fig. 2 Comparison of Controlled angles

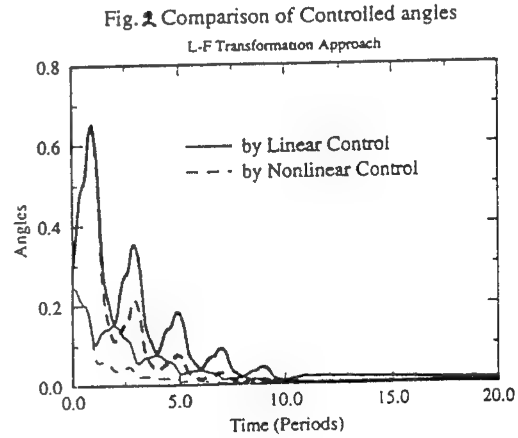
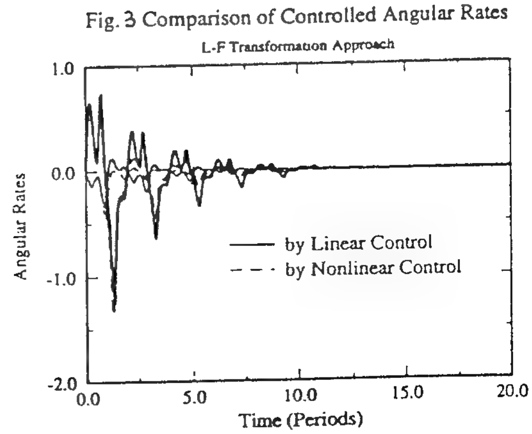


Fig. 3 Comparison of Controlled Angular Rates



SPURIOUS SOLUTIONS PREDICTED BY THE HARMONIC BALANCE METHOD

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The harmonic balance (HB) method is commonly used to analyze the steady state periodic response of both weakly and strongly nonlinear systems. In this method a periodic solution for the independent variable 'x' is assumed as a truncated Fourier series plus a residual term 'v', i.e.,

$$x = x_{\circ} + v = r_{\circ} + \sum_{i=1}^{n} r_1 \cos(i\Omega t - \phi_i) + v. \tag{1}$$

This solution, or any one of its alternate forms, is substituted in the equation(s) of motion, and the coefficients $F_{\circ}$, F_j, G_j, $j = 1, \cdots, n$, of the bias and the harmonics $\cos(j\Omega t)$, $\sin(j\Omega t)$, respectively, are equated to zero. The resulting coupled, nonlinear algebraic equations are solved simultaneously to obtain the parameters $r_{\circ}$, r_i, and ϕ_i used in the approximate solution $x_{\circ}$ for the steady state periodic response. Other terms provide the so-called "variational equation", which is an exact, transformed equation for the variational displacement v. An HB solution in which only a few leading harmonics are used can fail to predict the existing periodic solutions (type 1 failure) and/or it may predict periodic solutions which have no counterpart in the actual response (type 2 failure).

It is clear that a possible sub- or superharmonic resonance can not be detected by an HB approximation unless at least some of the appropriate harmonics are included in the assumed solution. Therefore, examples of type 1 failure are abundant. The case of second superharmonic resonance in the Duffings oscillator, $\ddot{x} + \delta\dot{x} + x + x^3 = F\cos(\Omega t)$ (eq 2), provides a simple example for which a higher order HB solution exhibits both type 1 and type 2 failure.

The results for eq. 2, with $F = 20$ and $\delta = 0.2$, are shown in fig. 1. For clarity, only the amplitude of the fundamental harmonic is shown. The results for the third superharmonic resonance, shown by a solid line, were determined by using $n = 3$ in (1). The results for the second ($\triangle$) and the third superharmonic ($\square$) resonance obtained by using $n = 15$ in (1) are also shown. These higher order HB results are essentially identical to the numerical results. The results for the second superharmonic resonance are shown by a dotted line. These HB results, shown in figs. 1(a) and 1(b), were determined by using $n = 3$ and $n = 4$ in (1), respectively. Hatch marks indicate unstable solutions. The stability type of the predicted solutions was determined by an approximate analysis of the corresponding variational equation.

Clearly, the HB solution for the second superharmonic resonance based on $n = 3$ (fig. 1(a)) a) erroneously predicts that the pitchfork bifurcation at point B is subcritical, b)

erroneously predicts the existence of a saddle-node bifurcation at point G, c) predicts the fictitious branch B–G for the unstable solutions of the second superharmonic resonance, and d) fails to predict the stable solutions for the second superharmonic resonance for $\Omega > \Omega_G$.

The HB approximation obtained by using $n = 4$ in (1) provides qualitatively correct frequency-response curves for the second superharmonic resonance (fig. 1(b)). Clearly, the qualitative failure of the harmonic balance method is restricted neither to a first approximation nor to the systems having asymmetric potential well. It has also been checked that the residual values for some of the neglected higher harmonics can not detect a qualitative failure of the HB method.

In symmetry breaking and period doubling pitchfork bifurcations, some new harmonics are brought into the system response. Close to the bifurcation point(s), these harmonics have small amplitudes, which are comparable to each other. Some of these harmonics are instrumental in deciding the sub- or supercritical nature of the pitchfork bifurcation. The importance of their relative role in the bifurcation can not be decided a priori. The harmonic balance method can lead to erroneous results if one or more of these small but instrumental harmonics are neglected from the assumed solution. This result does not agree with the common belief that the HB method provides qualitatively correct results when "all" dominant harmonics are included in the assumed solution. In the periodic response of strongly nonlinear systems, certain harmonics which are more (respectively, less) significant in a given domain of the system parameters may become less (more) significant in a different region of the parameter space. This makes the problem of finding the right mix of the few leading harmonics, which will provide qualitatively correct response curves, even more difficult. Further details are given in [1].

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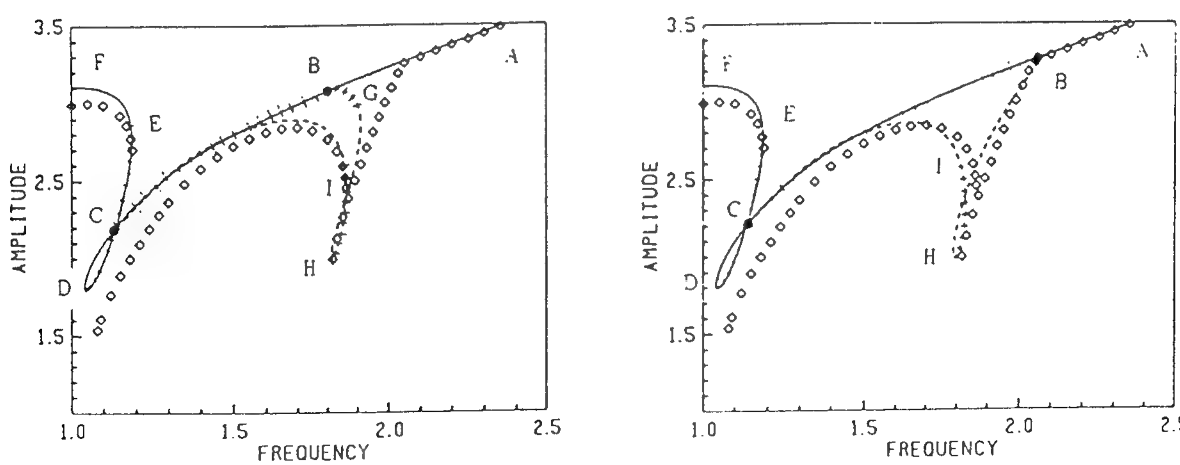

Figure 1(a)   Figure 1(b)

A RENOVATED ALGORITHM FOR INCREMENTAL HARMONIC BALANCE METHOD

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Abstract

The main difference between a linear system and a nonlinear system is in the non-uniqueness of solutions manifested by the singular Jacobian matrix. It is important to be able to express the Jacobian accurately, completely and efficiently in an algorithm to analyze a nonlinear oscillator. For periodic response, the incremental harmonic balance (IHB) method is widely used. The existing IHB methods, however, requiring double summations of each harmonic components to form the Jacobian matrix, are often extremely time-consuming when higher order harmonic terms are retained to fulfill the completeness requirement. A new algorithm to compute the Jacobian is to be introduced with the application of fast Fourier transforms (FFT) and Toeplitz formulation. The resulting Jacobian matrix is constructed explicitly by three vectors in terms of the current Fourier coefficients of response, depending respectively on the synchronizing mass, damping and stiffness functions. The part of the Jacobian matrix depending on the nonlinear stiffness is actually a Toeplitz matrix. The other parts of the Jacobian matrix depending on the nonlinear mass and damping are Toeplitz matrices modified by diagonal matrices. If the synchronizing mass is normalized

in the beginning, we need only two vectors to construct the Toeplitz Jacobian matrix (TJM). The present method of TJM is found to be superior in both computation time and storage than all existing IHB methods due to the simplified explicit analytical form and the use of FFT.

The aim of the present paper is to introduce a new computational algorithm with the application of FFT technique and Toeplitz formation. It is capable to substantially reduce the amount of computational work involved. Based on the concept of Galerkin averaging theory and discrete Fourier transformation, we provide an explicit formula for the Jacobian matrix and suggest an efficient solution path. A fast Fourier transform algorithm is then applied to compute the Jacobian, the (irrational) nonlinearities and the Floquet exponents (for stability checking). The procedure provided in present paper can easily be extended to subharmonic response of multiple DOF system with general forms of nonlinearities.

Table I Comparision the Numbers of Multiplication

Method	Number of Multiplications
Direct IHB	$11M(2N+1)^2$
Present	$11(2N+1)^2 + 2M\log_2 M$

Table II Numerical Comparasion of RIHB vs. IHB

(for step length 0.4 and residual tolerence 1.e-6)

No.of Step	Frequency Ratio ω (1/rad)	1st Order Amplitude	Number of Iterations I and CPU - times T				
			IHB method (a)		Present Method (b)		
			I_a	T_a	I_b	T_b	T_b/T_a
0	1.000000	3.935953	12	5.65	5	0.49	8.67%
1	1.173276	4.343391	12	6.21	4	0.55	8.86%
2	1.302025	4.676480	16	8.18	5	0.66	8.06%
3	1.419599	5.000948	18	9.18	5	0.61	6.64%
4	1.528153	5.315249	22	11.15	5	0.61	5.47%
5	1.628985	5.617792	29	14.67	5	0.65	4.43%
6	1.722977	5.907414	36	18.07	5	0.66	3.65%
7	1.810760	6.183333	42	21.09	5	0.66	3.13%
8	1.892812	6.445093	46	23.01	5	0.61	2.65%
9	1.969521	6.692505	49	24.49	5	0.61	2.49%
Total Running Time				2:46.48		0:06.21	3.67%

Table III. Stability points along solution path (fig.2)

f	Stability	Property	η	λ
0.000000	1x1T stable			
0.461353	1x1T unstable	Fold		
0.228323	1x1T stable			
2.395560	2x1T stable	Symmetric Breaking		
3.291727	2x1T untable	Fold		
3.257349	2x1T stable		2.9915	3.41
5.387085	4x2T stable	Period 2 Bifurcation		
6.264048	8x4T stable	Period 4 Bifurcation	0.8769	4.39
6.443806	16x8T stable	Period 8 Bifurcation	0.1998	4.56
6.483227	32x16T stable	Period 16 Bifurcation	0.0394	

η —— distance between period double bifurcation points

λ —— ratio of distance

Table IV Comparison of CPU Times

ν	TERMS	PRESENT METHOD	R.K. Method	
			Regular P.T	Singular P.T(f)
1	21	00:00.44	00:18.13	03:23.44(2.395561)
2	41	00:10.10	00:22.19	21:35.15(5.387086)
4	81	00:16.75	00:25.98	06:53.76(6.264049)
8	161	00:50.41	00:48.90	03:03.83(6.443807)

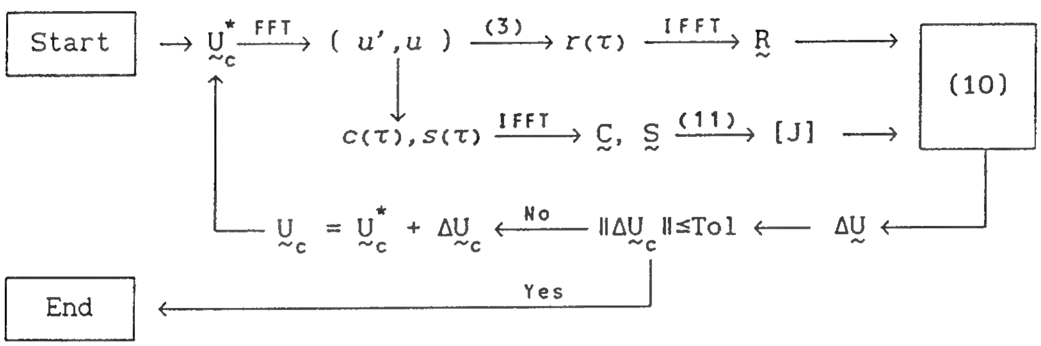

Fig.1 Flow chart for Newtonian Iteration
(* represents the unbalanced state)

ON THE ACCURACY OF THE "SELECTED BLOCK" APPROACH TO THE LOCAL STABILITY ANALYSIS OF THE APPROXIMATE HARMONIC BALANCE SOLUTIONS

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In the harmonic balance (HB) method, an approximate solution for the periodic response is assumed as a truncated Fourier series plus a residual term. The approximate solutions for the periodic steady state response are determined from the resulting coupled, nonlinear algebraic equations. Other terms, i.e., those involving higher harmonics or the residual, provide the so-called variational equation. The stability type of the approximate HB solutions is determined by analyzing the linearized variational equation (LVE). This stability analysis is usually carried out by using a second harmonic balance step. By using the Floquet theory and an infinite Fourier expansion in the harmonic balance method, one obtains an infinite set of linear homogeneous equations for the parameters used in the assumed Fourier expansion. The matrix of coefficients for these parameters is used to determine the stability type of the approximate solutions.

In most cases, the sign of the determinant of the leading 2n×2n or (2n+1)×(2n+1) matrix, n=$1, 2, \cdots$, is used for this purpose. This is equivalent to using a finite number of terms in the Fourier series expansion of the periodic component of the assumed solution for the LVE. However, sometimes the determinant of the 2×2 block which corresponds to a specific harmonic in the assumed solution for the LVE is used for this purpose. This "selected block" approach for the local stability analysis is equivalent to using a single, selected harmonic in the assumed solution for the LVE.

This approach is usually employed when the specific selected harmonic is absent from the periodic solution for which an HB approximation is being established, but this harmonic is expected to play a dominant role in the periodic solutions which bifurcate from the approximate solution. Here, three examples are presented to show that the stability information obtained by this selected block approach can contain significant errors.

The approximate HB solution for the primary resonance in the Duffings oscillator,

$$\ddot{x} + \delta\dot{x} + x + x^3 = F\cos(\Omega t),$$

for $F = 20$ and $\delta = 0.2$, are shown in fig. 1. These results were determined by using the harmonics 1 and 3 in the assumed HB solution. The stability results determined from the selected block based on the fundamental component in the assumed solution of the corresponding LVE are also shown. The hatched line indicates unstable solutions. Clearly, these results contain significant errors.

The HB results for the third superharmonic resonance in eq. 1 are shown in fig. 2. These results were determined by using the harmonics 1 and 3 in the assumed solution.

The stability type of these solutions determined from the selected block based on the third superharmonic component in the assumed solution of the LVE are also shown. Clearly, these stability results are qualitatively incorrect.

The stability information for the possible buildup of even harmonics in the above third superharmonic response is shown in fig. 3. This stability information was established by using the 2×2 block based on the second superharmonic component in the assumed solution for the LVE. These results indicate that the third superharmonic response is unstable, and the second superharmonic resonance can excited in (1), in the frequency band $1.86 \leq \Omega \leq 2.26$. The actual numerical results for the steady state response of (1) are also shown. These numerical results correspond to the primary, 2nd and 3rd superharmonic, and the 5/2 subharmonic resonances. These numerical results show that the second superharmonic resonance is excited for $1.154 \leq \Omega \leq 2.09$. Again, the stability results determined from the "selected block" approach contain significant errors. Further details will be presented at the conference.

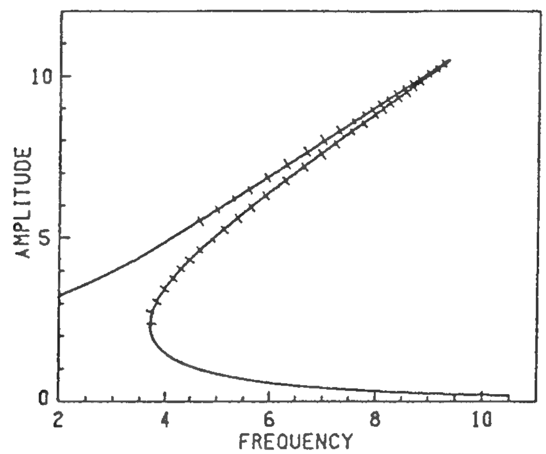

Figure 1.

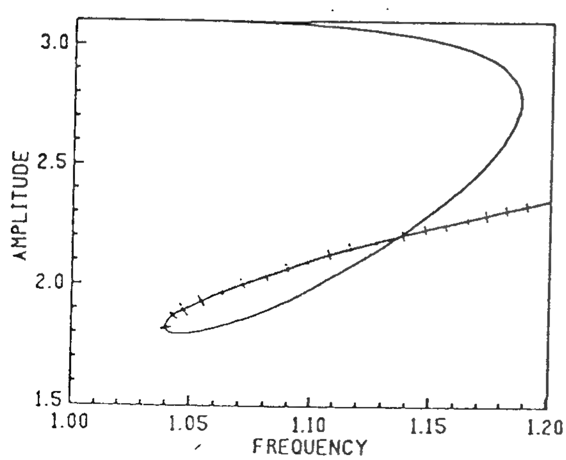

Figure 2.

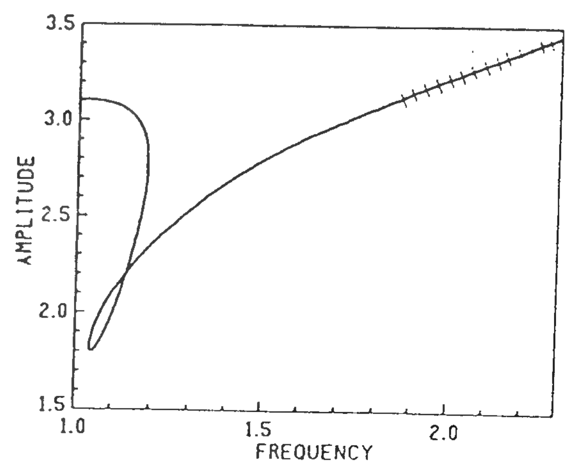

Figure 3.

Lie-Trasformation Method for Dynamics and Control of Weakly Nonlinear Autonomous Systems

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The paper presents recent developments in a singular perturbation method, known as the Lie transformation method for the analysis of weakly nonlinear autonomous dynamical systems (specifically, first-order differential equations with algebraic nonlinearities): a general theory for obtaining and discussing an analytical response for the solution of a system undergoing to a local bifurcation is introduced; moreover, using a very simple nonlinear control law, nonlinear-driving terms are suitable evaluated in order to ensure that beyond stability boundary (Hopf bifurcation) the system exibits a stable-limit-cycle behavior (rather than an unstable one).

The Lie transformation method was introduced by Deprit [1], further developed by Kamel [2] (see also in Nayfeh [3] and Lichtenberg and Lieberman [4]). Morino, Mastroddi, and Cutroni [5] and Mastroddi [6] applied the method to the analysis of dynamical systems with algebraic nonlinearities (see equation (1)). Within this context they demonstrates that the method predict correctly non-simple-harmonic limit cycles or chaotic behavior. Specifically, in the neighborhood of the Hopf bifurcation, an approximate solution (with an error of an order of magnitude of the fourth power of the limit cycle amplitude) is obtained for the transient response of the system. The nature of this solution is determined by the nature of the non linear terms, which is characterized (stable or unstable cycle limite, unstable or unstable fixed point, etc...) by one single real number $\gamma_R^{(1)}$; therefore, one might use a nonlinear-control law, by adding a suitable nonlinear term so as to modify the parameter $\gamma_R^{(1)}$ in order to obtain stabilizing effects.

In order to clarify the aim of the present paper, we briefly outline the theory presented in Ref. [5]. For simplicity we limit ourselves to the case of a Hopf bifurcation (the formulation for more complex responses, in particular for chaotic behavior, is very similar to that presented below). Consider a dynamical system with nonlinearities of polynomial nature (suppose cubic for the sake of simplicity)

$$\dot{\mathrm{x}} = (\Lambda + \varepsilon \mathrm{A})\mathrm{x} \; + \; \varepsilon \left\{ \sum_{p,q,r=1}^{N_S} c_{jpqr} \; x_p x_q x_r \right\} \; + \; \ldots \tag{1}$$

(ε small parameter) where N_S is the dimension of the space-state vector, Λ is a diagonal matrix with diagonal terms λ_j, and A is a full matrix. We assume that all the eigenvalues are complex conjugate, all with negative real part except for the first pair which is purely imaginary (similar results can be presented also in the presence of real eigenvalues). We research for a solution is given in terms of an asymptotic expansion of the type

$$\mathrm{x} = \check{\mathrm{x}}(\mathrm{y}, \varepsilon) = \mathrm{y} + \varepsilon \check{\mathrm{x}}_1(\mathrm{y}) + \mathcal{O}(\varepsilon^2) \tag{2}$$

which is referred to as the Lie transformation. Using this arbitrary transformation (note that $\check{\mathrm{x}}_1$ is an unknown vector), and setting

$$c_{jpqr} = c^{\mathsf{E}}_{jpqr} + c^{\mathsf{A}}_{jpqr} \qquad\qquad a_{jl} = a^{\mathsf{E}}_{jl} + a^{\mathsf{A}}_{jl} \tag{3}$$

where either c^{E}_{jpqr} and a^{E}_{jl} or c^{A}_{jpqr} and a^{A}_{jl} equals zero, one obtains (see Ref. [5])

$$\check{x}_{1_j} = \sum_{l=1}^{N_S} \frac{1}{\lambda_l - \lambda_j} a^{\mathsf{A}}_{jl}\, y_l + \sum_{p,q,r=1}^{N_S} \frac{1}{\lambda_p + \lambda_q + \lambda_r - \lambda_j} c^{\mathsf{A}}_{jpqr}\, y_p y_q y_r \tag{4}$$

and equation (1) yields

$$\dot{\mathbf{y}} = \Lambda \mathbf{y} + \varepsilon \left\{ \sum_{l=1}^{N_S} a^{\mathsf{E}}_{jl} y_l \right\} + \varepsilon \left\{ \sum_{p,q,r=1}^{N_S} c^{\mathsf{E}}_{jpqr} y_p y_q y_r \right\} + \ldots \tag{5}$$

It is apparent that the *essential* terms (a^{E}_{jl} and c^{E}_{jpqr}) are those that contribute to the $\mathbf{y}$-problem, whereas the *auxiliary* ones (a^{A}_{jl} and c^{A}_{jpqr}) appear in the $\check{\mathbf{x}}_1$-problem. The subdivision into *essential* and *auxiliary* terms stated in equation (3) is to be performed with the objective that (see equation (4)) the terms such that $\lambda_l - \lambda_j = 0$ and $\lambda_p + \lambda_q + \lambda_r - \lambda_j = 0$ (zero divisors) must be classified as *essential* terms (these terms correspond to those typically referred to as long-period terms, in the scientific literature, see, *e.g.*, Refs. [1] to [3]). Within the above choice, the equation (5) has the solution (see also Ref. [6])

$$y_1 = \left[\frac{-\beta^{(1)}_R / \gamma^{(1)}_R}{1 + k\; e^{2\beta^{(1)}_R t}} \right]^{1/2} e^{i\varphi_1} \tag{6}$$

$$y_n = a^0_n \left(\frac{a_1}{a^0_1} \right)^{\gamma^{(n)}_R / \gamma^{(1)}_R} e^{t\,(-\beta^{(n)}_R + \beta^{(1)}_R \gamma^{(n)}_R / \gamma^{(1)}_R)}\; e^{i\varphi_n} \qquad n = 3, 5, \ldots, N_s - 1 \tag{7}$$

($y_2 \equiv y_1^*$, $y_4 \equiv y_3^*$, ..., $y_{n+1} = y_n^*$) where $\varphi_n := (-\beta^{(n)}_I + \gamma^{(n)}_I \beta^{(1)}_R / \gamma^{(1)}_R) + (\gamma^{(n)}_I / \gamma^{(1)}_R) \ln(a_1) + \varphi^0_n$ and where $\beta^{(n)} = \beta^{(n)}_R + i\beta^{(n)}_I$ and $\gamma^{(n)} = \gamma^{(n)}_R + i\gamma^{(n)}_I$ ($n = 1, 3, \ldots, N_S - 1$) are coefficients depending on linear and nonlinear coefficients (a_{jl} and c_{jpqr}) respectively and k, a^0_n, and φ^0_n on the initial conditions.

Applications to the problem of the flutter of a wing in supersonic flow (with algebraic nonlinearities arising from the aerodynamic description Ref. [6]) are included: the results indicate that it is possible to achieve stabilization of the unstable limit cycle, which means that destructive flutter is replaced with benign flutter. Additional applications, in particular control of chaotic response, will be esamined.

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Wednesday, June 15

1030-1210

Session 12. Analytical Methods II

CONSTRUCTING GALERKIN'S APPROXIMATIONS OF INVARIANT TORI USING MACSYMA

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Invariant tori of solutions for nonlinearly coupled oscillators are generalizations of limit cycles in the phase plane. They are surfaces of aperiodic solutions of the coupled oscillators with the property that once a solution is on the surface it remains on the surface. This paper is a case study of the computational experience involved in applying both symbolic and numerical methods to the construction of analytic representations of invariant tori for a larger range of the forcing terms.

The construction of a parametric representation of an invariant torus for a system of coupled oscillators of the form

$$\begin{aligned}\ddot{x}_1 + \omega_1^2 x_1 &= \epsilon f_1(\mathrm{x}, \dot{\mathrm{x}}),\\ \ddot{x}_2 + \omega_2^2 x_2 &= \epsilon f_2(\mathrm{x}, \dot{\mathrm{x}}),\end{aligned} \tag{1}$$

where $\mathrm{x} = (x_1, x_2)^T$, $\dot{\mathrm{x}} = (\dot{x}_1, \dot{x}_2)^T$, $\epsilon > 0$, can be reduced to the construction of a solution of a system of partial differential equations. By introducing polar coordinates (1) can be reduced to a system of the form

$$\begin{aligned}\dot{\theta} &= \mathrm{d} + \epsilon\Theta(\theta, \mathrm{x}),\\ \dot{\mathrm{x}} &= \epsilon \mathbf{X}(\theta, \mathrm{x}),\end{aligned} \tag{2}$$

where $\theta = (\theta_1, \theta_2)^T$, $\mathrm{d} = (1,1)^T$, $\Theta = (\Theta_1, \Theta_2)^T$, $\mathrm{x} = (x_1, x_2)^T$, $\mathbf{X} = (X_1, X_2)^T$. $\Theta(\theta, \mathrm{x})$, $\mathbf{X}(\theta, \mathrm{x})$ are assumed to be periodic with vector period $2\pi/\omega = (2\pi/\omega_1, 2\pi/\omega_2)$.

A parameterized surface $\mathrm{x} = \mathrm{S}(\theta)$, with vector period $2\pi/\omega$, is an invariant torus for (2) if given that $\theta(t)$ solves

$$\dot{\theta} = \mathrm{d} + \epsilon\Theta(\theta, \mathrm{S}(\theta)), \tag{3}$$

for all $t \in (-\infty, \infty)$, then $(\theta(t), \mathrm{S}(\theta(t)))^T$ solves (2) for all $t \in (-\infty, \infty)$. It is not hard to show that $\mathrm{S}(\theta)$ must satisfy the system of partial differential equations

$$(N\mathrm{S})(\theta) = D\mathrm{S}(\theta) \cdot (\mathrm{d} + \epsilon\Theta(\theta, \mathrm{S})) - \epsilon\mathbf{X}(\theta, \mathrm{S}) = \mathbf{0}, \tag{4}$$

where

$$D\mathrm{S}(\theta) = \left(\frac{\partial S_i}{\partial \theta_j}\right)_{i,j=1,2}. \tag{5}$$

For a Galerkin approximation assume a trial solution of the form

$$\mathrm{S}_K = c_1\phi_1(\theta) + \cdots + c_K\phi_K(\theta), \tag{6}$$

where $\{\phi_i(\theta)\}$ is a basis set, each $\phi_i(\theta)$ periodic with vector period $2\pi/\omega$. The parameters $c_1, \cdots, c_K$ are selected to satisfy

$$\left\langle N(\sum_{i=1}^{K} c_i\phi_i(\theta)), \phi_j(\theta) \right\rangle = 0 \tag{7}$$

for $j = 1, \cdots, K$, where $\langle \cdot, \cdot \rangle$ is an appropriately defined inner product.

Once the trial approximation is substituted into (4) the intermediate series must be manipulated efficiently. This is accomplished by using an intermediate series representation called a Poisson series. The symbolic manipulative capabilities of this series representation are available in MACSYMA. The Poisson series is a special form of a multiple Fourier series that can be compactly represented in a computer's memory using linked list representations. In particular only such items as the type of each term (sine or cosine), the coefficients of the angular terms, exponents of polynomials and term coefficients need be stored. Using linked lists allows rapid addition and multiplication of series since there are very efficient algorithms for adding and multiplying linked lists.

Two applications to Van der Pol systems of the Poisson series subpackage of MACSYMA as an intermediate tool to computing the Galerkin approximation of invariant tori were studied. For the classic Van der Pol oscillator three cases were considered. They were for $\epsilon = 0.5, 1.0, 1.5$. For the case $\epsilon = 0.5$ a constant term and the first 7 even harmonic terms were computed. The approximation errors were 3e-4 for the angular variable and 5e-4 for the radial variable. For the case $\epsilon = 1.0$ a constant term and the first 9 even harmonic terms were computed. The approximation errors were 8e-3 for the angular variable and 9e-3 for the radial variable. Finally for the case $\epsilon = 1.5$ a constant term and the first 12 even harmonic terms were computed. The approximation errors were 0.1 for the angular variable and 0.2 for the radial variable. The errors could have been reduced by adding more terms to the Galerkin approximation.

For the case of coupled Van der Pol oscillators the computational experience was similar but much more extensive. Each harmonic added to the Galerkin representation added many terms to the basis set representations. Thus only a limited number of harmonic terms could be included for computation on a PC. For the three cases considered, $\epsilon = 0.05, 0.5, 1.0$, only the constant terms and the first 2 even harmonics were included. In each of these cases though this required solving for 50 coefficients. For the case $\epsilon = 0.05$ the maximum angular error was approximately 5e-3 and maximum radial error was 2e-3. For the case $\epsilon = 0.5$ the maximum angular error was approximately 0.9 and the radial error was 1.5. Finally for the case $\epsilon = 1.0$ the maximum error was approximately 10.0 and the radial error was 5.0.

This computational case study demonstrated that the symbolic manipulation capabilities of MACSYMA provided sufficient tools to compute invariant tori. The accuracy of the approximations was primarily limited by the available useful computer memory. Processor speed also made it possible to solve for a large number of coefficients in a reasonable amount of time.

The Dynamics of Resonant Capture

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Abstract

Resonant capture describes the behavior of a weakly coupled multi–degree–of–freedom system when two or more of its uncoupled frequencies become locked in resonance. Flow on the region of phase space near the resonance (the resonance manifold) involves a region bounded by a separatrix in the uncoupled ($\epsilon = 0$) system. Capture corresponds to motions which appear to cross into the interior of the separated region for $\epsilon > 0$.

We offer two approximate methods for estimating which initial conditions lead to capture: an energy method and a perturbation method based on invariant manifold theory. These methods are applied to a model problem involving the spinup of an unbalanced rotor attached to an elastic support.

Nonlinear Parametric Identification by Balancing Harmonics of Extracted Periodic Orbits

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Abstract

We identify parameters of the differential equations of motion representing a chaotic system. Unstable periodic orbits are extracted from a chaotic set. The method of harmonic balance is applied to these periodic orbits to identify the system parameters.

Introduction. Parametric identification of nonlinear systems has been studied, for example, in [1-10]. Yasuda *et al.* [8-10] did this by applying the harmonic-balance method to periodic responses. We apply this idea to chaotic responses simply by extracting unstable periodic orbits. Other methods (e.g. [4-6]) are also applicable to chaotic data.

It may be natural to work with chaotic sets. Supposed we are interested in describing a distributed, infinite-dimensional system with a low-dimensional model. Chaos provides a vehicle for a dimensionality study from a small number of observed quantities [11-13], resulting in an estimate on the number of active state variables. Given the number of state variables, the low-dimensional modeling might typically involve identifying parameters in a set of differential or difference equations. An example of a paper which makes a bridge between these levels of the modeling process is that of [14].

In this study, we present numerical results on several different types of forced oscillators. The idea is also applicable to autonomous systems.

Harmonic Balance in Parametric Identification. We illustrate the idea [8-10] for a single-degree-of-freedom oscillator in the form

$$\ddot{x} + \alpha_1 f_1(x, \dot{x}) + \cdots + \alpha_p f_p(x, \dot{x}) = a\cos(\omega t). \tag{1}$$

If $x(t)$ is a periodic solution, then it can be approximated by a truncated Fourier series, i.e. $x(t) \approx x_o + \sum_{i=1}^{i=m} a_i \cos(i\omega t) + b_i \sin(i\omega t)$. Suppose x and $\dot{x}$ are sampled. Then any function of $f_j(x, \dot{x})$ can be performed for each sample, and is periodic. Thus, $f(x, \dot{x})$ can be approximated by a truncated Fourier series. The Fourier coefficients are computed for each term. The truncated Fourier series of $\ddot{x}$ can be obtained from that of $\dot{x}$. All of these terms are inserted into equation (1), and the harmonic coefficients are balanced. The result is $2m + 1$ linear equations in the p unknown coefficients, in the form $\mathbf{A}\alpha = \mathbf{b}$, where α is a vector of unknown parameters. If $2m + 1 > p$, the coefficients can be estimated through a least-squares fit. (This simple linearity is a much better scenario than what the harmonic-balance method produces in forward analysis.) Truncation of the Fourier series expansions simply limits the redundancy.

Periodic-Orbit Extraction. Since the harmonic-balance method calls for periodic data, we extract such data from a chaotic set [15-17]. This is possible since a chaotic

attractor is the closure of infinitely many unstable periodic orbits; a trajectory is constantly visiting periodic orbits. Although these periodic orbits are unstable, they are indeed solutions, and should satisfy the equations of motion. For a sufficiently large time series $\mathbf{x}_{i=1,\ldots,N}$, with n_o points per period, the trajectory will visit an unstable orbit of period T at some sample time j. If, after $K = n_o T$ iterations the trajectory points $\mathbf{x}_j$ and $\mathbf{x}_{j+K}$ are within some small distance ϵ, such that $||\mathbf{x}_j - \mathbf{x}_{j+K}|| < \epsilon$, then all the samples in between are considered to be near a period-T orbit. As a rule of thumb, $\epsilon = 0.005$ works well for normalized data [15-17].

One trajectory may produce several approximated periodic orbits. If there are n orbits extracted, we have $(2m+1)n$ equations in p unknowns, increasing the redundancy of the fit. Subharmonics may add more equations. Increased redundancy may be beneficial for systems with many unknown parameters.

Discussion. The idea has been tested on smooth single- and multi-degree-of-freedom oscillators, parametrically forced systems, and Coulomb-damped systems. There is some trouble with the multivaluedness of discontinuous damping functions during stick slip. This arises because the assumed functions do not incorporate the multivalued characteristic.

This preliminary study suggests that the harmonic balance method may be a easily implemented tool in parametric system identification of chaotic systems. There are further steps to be taken both in analysis of the technique, and in experimental implimentation. Analyses could be performed on sensitivity to error in conjuction with the approximate results of the periodic extraction. An experimental study is underway. Other questions regard the choice of assumed nonlinearities, and dealing with a limited number of sampled quantities.

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Using Adjoint Operator Method to Compute Normal Form of Order 4 For Nonlinear Dynamical System

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Abstract:

Normal form theory is a very strong method when we study degenerate bifurcations of nonlinear dynamical systems. In this paper we use adjoint operator method to compute normal forms of order 3 and 4 for nonlinear dynamical system with nilpotent linear part and Z_2—asymmetry. According to normal forms that we obtain, we study universal unfoldings for some degenerate bifurcation cases of codimension 3 and simple global characterizations.

In order to study bifurcations of nonlinear dynamical systems in the degenerate cases of higher codimension number (>3), we must compute higher normal forms of nonlinear dynamical systems. In recent twenty years many scientists made very important contributions to development of nonmal form theory, for example Arnold [1], Bogdanov [2,3], Bruno [4], Chow and Hale [5], Chow and Wang [6], Cushman and Sanders [7,8], Cushman et al [9], Elphick and Iooss et al [10] Guckenheimer and Holmes [11], Rand and Armbruster [12], Takens [13,14]. At the same time some scientists utilized theory of normal form and universal unfolding to study degenerate bifurcations of codimension 2 and global bifurcations in nonlinear oscillators, for example Holmes and Rand [15], Holmes [16], Bajaj [17,18], Show et al [19], Namachchivaya [20] Zhang et al [21,22,23].

The studies on degenerate bifurcations of codimension 3 and 4 have obtained very great developments. Dumortier et al [24] studied the cusp case of codimension 3.In [25] Dumortier et al studied degenerate bifurcations of codimension 3 and unfoldings of saddle, focus and elliptic singularities. In the literature [26] Dumortier et al used Macsyma and Mathematica to compute normal forms of quadratic models and studied degenerate bifurcations of codimension 3 and 4 of vector fields on the plane. Later Li et al [27] further study the cusp case of codimension 4. Joyal [28] studied the cusp case of order N. In [29] Li et al studied degenerate bifurcations of codimension 3 form of 1:2 resonance. Dangelmayr and Guckenheimer [30] studied a four parameter family of planar vector fields and codimension 3 and 4 bifurcations and used Macsyma to compute normal forms.

Adjoint operator method presented by Elphick and Iooss et al [10] is one of three ba-

sic methods of computing normal forms. The other two methods are matrix representation method [11,31] and method based on representation theory of Lie algebra sl(2, R) presented by Cushman and Sanders [7]. When we compute higher order normal forms, compared with matrix representation method, adjoint operator method has advantage that we do not have to compute repeatedly higher order matrices and larger linear algebraic equations. Therefore calculating work by adjoint operator method is less than that by matrix representation. In the literature [32] we used matrix representation to compute normal form of order 5 of nonlinear dynamical system with Z_2—symmetry.

In this paper we study nonlinear dynamical system with Z_2—asymmetry

$$\begin{cases} \dot{x}_1 = x_2 + \sum_{j=1}^{2} a_{1j}^{(0)}\mu_j + \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}^{(1)}\mu_j x_i + \sum_{i+j=2}^{4} a_{ij}x_1^i x_2^j \\ \dot{x}_2 = \sum_{j=1}^{2} b_{1j}^{(0)}\mu_j + \sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij}^{(1)}\mu_j x_i + \sum_{i+j=2}^{4} b_{ij}x_1^i x_2^j \end{cases} \tag{1}$$

where $\sum_{i+j=2} a_{ij}x_1^i x_2^j = a_{20}x_1^2 + a_{11}x_1x_2 + a_{02}x_2^2$, other terms are similar to this representation, and all coefficients are real.

When $\mu_j = 0$, j=1, 2, the zero solution of system (1) has double zero eigenvalues, that is, linear part of system (1) is nilpotant. Therefore we have

$$A = D_x X(x)|_{x=0} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{2}$$

In the following we use adjoint operator method to compute normal forms of order 3 and 4 for system (1) and study universal unfoldings in some codiemnsion 3 degenerate bifurcations and some simple global characterizations.

Improving the equivalent linearization for stochastic Duffing oscillator

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In stochastic nonlinear dynamics one is usually content with seeking a statistical description of the lowest order moments of mean and covariance. Under such a limited goal, it is often possible to replace the original nonlinear system by a surrogate linear system and thereby attempt to replicate the statistical dynamics inasmuch as the mean and covariance are concerned. This is the intent of the so-called statistical linearization or quasi-linearization. In particular, when the nonlinear system under consideration is an oscillator, it is also called statistical equivalent linearization, conforming with such a terminology already introduced by Krylov and Bogoliubov for the determinstic nonlinear oscillator. The statistical linearization was first introduced independently in 1953 by Caughey for nonlinear oscillators and by Booton for nonlinear circuits, both subjected to the Gaussian white noise excitation . It is however interesting to point out that several years later Bellman and Richardson proposed a self-consistent solution of the stochastic nonlinear differential equations, which essentially embodies elements of the statistical linearization. Since a randomly driven Duffing oscillator will be discussed in this paper, we shall restrict ourselves here to the term "equivalent linearization".

The investigation of this paper was initiated by an attempt to assimilate the widely different error estimates on the response variance of Duffing oscillator. Lyon was the first to show that the equivalent linearization underestimates the response variance and the maximum error is less than 10%. Some twelve years later, by repeating the error estimation of Lyon, Iwan and Yang concluded that the equivalent-linearization error is less than 7.5%, even in the case of arbitrarily large nonlinearity. However, their maximum 7.5% error is only a half the analytical upper error bound of 14.6% reported by Atalik and Utku, based on a Duffing oscillator without the linear stiffness term. Concurrently, Budgor et al. have presented some typical errors of the equivalent linearization in the range of 3.4% - 9.4%, without claiming to have found an upper error bound. It is somewhat surprising that these error estimates have been reported without making reference to each other's work. Hence, the first objective of this paper is to provide a unified framework to compare the various error estimates in statistical equivalent linearization.

As it turns out, assessing equivalent-linearization errors has led us to the source of this error. Hence, we can address rationally to a means for improving the equivalent linearization, which is the second objective of this paper. As shown by Budgor et al., the work involved in equivalent linearization is almost trivial compared to other perturbative procedures, yet it can provides a robust estimate on response variance for all but the very strong nonlinear case. The failing in such a case is due to the zero fourth-order cumulant assumption invoked so inconspicuously in the course of equivalent linearization. This could have been suspected from the stationary Fokker-Planck solution for Duffing oscillator, which is far from being Gaussian.

By using the fourth-order cumulant of the Fokker-Planck stationary distribution, one can recover the exact response variance by the usual equivalent linearization procedure. It therefore suggests estimating the fourth-order cumulant along with the response variance, and this has already been carried out by Crandall using the Edgeworth series representation for a non-Gaussian distribution. For completeness, we shall also discuss other methods for improving the statistical equivalent linearization by using certain weighting functions for the error minimization (Izumi et al.) and by seeking a higher order equivalent-linearization (Iyengar).

Because of the analytical moment expressions of Duffing oscillator, we are in a position to demonstrate certain mathematical identities, rather than simply offering conjectures based on numerics.

Wednesday, June 15

1330-1510

Session 13. Structural Dynamics I

APPROPRIATE STRESS AND STRAIN MEASURES FOR NONLINEAR STRUCTURAL ANALYSES

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In the modeling and analysis of structures undergoing large deformations and rotations, the stress and strain measures need to be work-conjugate, objective, geometric, and directional in order to use the experimentally-obtained material constants in the constitutive equation. Unfortunately, most strain measures are not objective, and some objective strains (invariant under rigid-body rotations; e.g., Green-Lagrange strains) are not geometric measures. Characteristics and objectivity of displacement gradients, Green-Lagrange strains, infinitesimal strains, engineering strains, first and second Piola-Kirchhoff stresses, Cauchy stresses, and engineering stresses are studied. Jaumann strains, which are defined by using the right stretch tensor from the polar decomposition of the deformation gradient tensor, prove to be objective geometric strains measured with respect to the deformed structural configuration.

Figure 1 shows the undeformed and deformed configurations of an infinitesimal element whose undeformed shape is a cube. Here, the frame xyz is an orthogonal curvilinear inertial frame, and the base vectors along the axes x, y, and z are denoted by $\mathbf{j}_1$, $\mathbf{j}_2$, and $\mathbf{j}_3$, respectively. The frame $\xi\eta\zeta$ represents the rigidly translated and rotated configuration of the frame xyz, and the base vectors along the axes ξ, η, and ζ are denoted by $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$, respectively. Moreover, $\mathbf{f}_k$ are forces acting on the deformed surfaces. We also use $\mathbf{v}(=\vec{Oo})$ to denote the absolute displacement vector of the point o and $\mathbf{u}(=0)$ to denote the displacement vector of the point o with respect to the frame $\xi\eta\zeta$. Since the elastic energy Π is due to relative displacements among material points, its variation $\delta\Pi$ can be represented in terms of relative displacements as

$$\begin{aligned}\delta\Pi &= \int_{V^0}(\mathbf{f}_1\cdot\mathbf{i}_n\delta\frac{\partial\mathbf{u}}{\partial x_1}dx_1\cdot\mathbf{i}_n+\mathbf{f}_2\cdot\mathbf{i}_n\delta\frac{\partial\mathbf{u}}{\partial x_2}dx_2\cdot\mathbf{i}_n+\mathbf{f}_3\cdot\mathbf{i}_n\delta\frac{\partial\mathbf{u}}{\partial x_3}dx_3\cdot\mathbf{i}_n)\\ &= \int_{V^0}J_{mn}\delta B_{mn}dV^0 \qquad (1)\end{aligned}$$

where the repeated subscript indices imply summations, $dV^0(=dx_1dx_2dx_3)$ denotes the undeformed system volume, and J_{mn} and B_{mn} are Jaumann stresses and Jaumann strains, respectively. It is shown that Jaumann strains and stresses can be easily derived by using a new concept of local displacements (without using the complex polar decomposition) as

$$B_{mn}=\frac{1}{2}(\frac{\partial\mathbf{u}}{\partial x_m}\cdot\mathbf{i}_n+\frac{\partial\mathbf{u}}{\partial x_n}\cdot\mathbf{i}_m)=B_{nm} \qquad (2)$$

$$J_{mn} = \frac{1}{2}\left(\frac{\mathbf{f}_m}{dx_p dx_q} \cdot \mathbf{i}_n + \frac{\mathbf{f}_n}{dx_r dx_s} \cdot \mathbf{i}_m\right) = J_{nm} \ , \quad m \neq p \neq q \ , \quad n \neq r \neq s \tag{3}$$

On the other hand, engineering strains ϵ_{mn} and engineering stresses σ_{mn} have the vector forms

$$\epsilon_{mn} = \frac{1}{2}\left(\frac{\partial \mathbf{v}}{\partial x_m} \cdot \mathbf{j}_n + \frac{\partial \mathbf{v}}{\partial x_n} \cdot \mathbf{j}_m\right) = \epsilon_{nm} \tag{4}$$

$$\sigma_{mn} = \frac{1}{2}\left(\frac{\mathbf{f}_m}{dx_p dx_q} \cdot \mathbf{j}_n + \frac{\mathbf{f}_n}{dx_r dx_s} \cdot \mathbf{j}_m\right) = \sigma_{nm} \ , \quad m \neq p \neq q \ , \quad n \neq r \neq s \tag{5}$$

We note that the vector form of Jaumann strains (Jaumann stresses) is the same as that of engineering strains (engineering stresses) except that $\mathbf{j}_m$ and $\mathbf{v}$ are replaced with $\mathbf{i}_m$ and $\mathbf{u}$. However, Jaumann strains are objective but engineering strains are non-objective. Fortunately, in experiments of measuring material constants by using engineering stress and strain measures, rigid-body motions are always prevented on purpose. Consequently, $\mathbf{j}_m = \mathbf{i}_m$ and $\mathbf{v} = \mathbf{u}$ and therefore engineering strains are objective in experiments. Hence, material constants obtained from experiments by using engineering stress and strain measures can be directly used in the constitutive equation of Jaumann stresses and strains, but not the constitutive equation of second Piola-Kirchhoff stresses and Green-Lagrange strains.

The use of Jaumann strains and stresses and a new concept of orthogonal virtual rotations in deriving geometrically-exact structural theories is fully illustrated in the formulation of thin shells having arbitrary initial curvatures and undergoing large displacements and rotations. Moreover, it is shown that energy and Newtonian approaches are fully correlated in the formulation and all energy terms can be interpreted in terms of vectors. Applications of Jaumann stresses and strains in the large-strain analysis of solids and composite structures will be discussed.

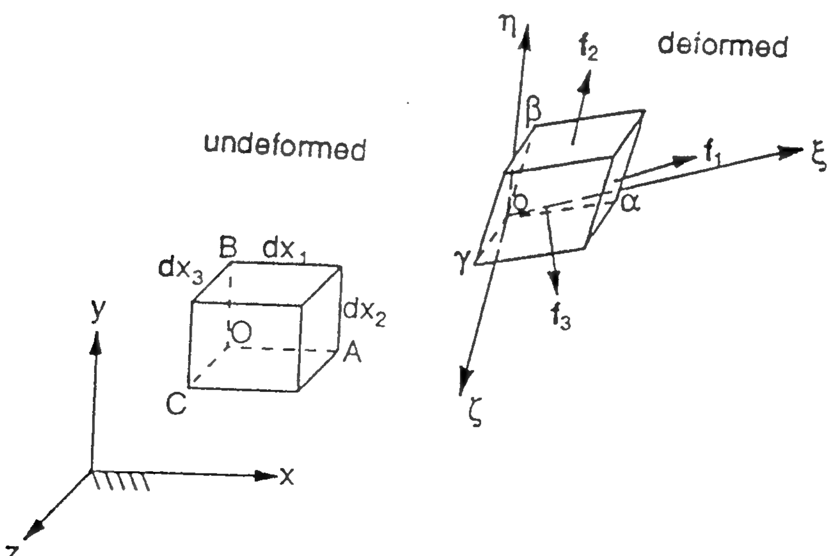

Fig. 1

SECONDARY SYSTEM ANALYSIS FOR SPACE PAY-LOAD

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Abstract

The aim of this paper is to study the behaviour of a secondary system carried by a space vehicle. The system is subjected to gravitational and fluid dynamical actions.

The global system (space vehicle and carried load) is modeled by a finite element tecnique. The paper provides suitable models for both the description of the fluid dynamical actions on the vehicle and the mechanical behaviour of its links (base isolators) with the secondary system.
The analysis consists of two different phases: the first is the representation of the different actions on the surface of the vehicle and their simulation in the numerical code. These actions depend on the velocity, the altitude of flight, the air density and the flight path of the flying body.

Introducing the velocity and the air density in given flight conditions, as input, the interaction between the space vehicle and the atmosphere is reduced to a vector of nodal forces (three for each node). They represent the actual input for the finite element analysis.

Several analyses representing different flight conditions provide the responses (acceleration, velocity and displacement) of the points of the vehicle where the base isolators of the pay-load are located. These responses are regarded as base motions of the secondary system toward its analysis.
The carried pay-load is linked with the vehicle by hysteretic elements introducing the nonlinearity of the problem.

The influence of the randomess affecting the base isolation properties is investigated by a stochastic finite element approach. The aim is to find an optimun mechanichal design of the links in order to minimize undesired accelerations of the carried system during different flying conditions.

Dynamic and Thermal Response of Space Payload Structures

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Dynamic responses of space payload structures subjected to lift-off environment on earth and to service environment in orbit are studied. While in a lift-off environment on earth, dynamic response of a space payload and its subsystems is analyzed due to high-amplitude external dynamic loadings or disturbances, such as lift-off forces and wind loads. In space, effects of extreme temperature variation, differential gravitational forces, in-house and external vibration causes are investigated on motion as well as deformation of a truss-like space structure and its subsystems (payloads attached to it). Throughout the study, particular attention is given to the effects of mass, frequency and damping ratios of secondary to primary system. The significance of the primary-secondary system interaction and mass ratio for near tuned conditions is also studied.

NONLINEAR FINITE ELEMENT BEHAVIOR OF COOLING TOWERS

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ABSTRACT

A three-dimensional full 360-degree finite-element model that is capable of realistically representing the response of two cooling towers at a nuclear power plant subjected to the plant design-basis safe shutdown earthquake, 90 mph wind, and 300 mph tornado is used to create a data pool which supports the proposed construction of a new safety-related facility in the shadow of these cooling towers.

The 300-mph tornado is made of a 240 mph tangential component, a 60 mph translational component and a negligible radial component.

During normal operation, power plants produce a substantial amount of heat that requires dissipation. The function of a natural draft cooling tower is to discharge the heat rejected from the turbine cycle to the atmosphere. Cooling towers are designed as doubly-curved hyperboloids to yield a higher buckling capacity than the more popularly used cylinders.

Each of the two cooling towers under study is 540-foot high with a radius at the base of the shell of 193.1', a radius at the throat of 141', and a radius at the top of 150.6'. The minimum thickness of the 4000-psi reinforced concrete shell is 8" with two layers of reinforcement. The shell thickness is varying at the base and top of the shell along the depth of the lower and upper lintel beams with a maximum value of about 3.1'. The towers are supported by unweathered foundation rock with a density of 170 lb/ft^3, a shear wave velocity of 6700 ft/sec, and a compressional wave velocity of 14000 ft/sec.

Figure 1 shows the finite element model of the cooling tower. The model consists of 1131 nodes, 80 column elements, and 1090 shell elements with membrane and bending capacities. Elements are selected to have the option of incorporating geometric nonlinearities and stress stiffening effects. The first three natural frequencies are found to be 1.70 and 2.61 Hz in the horizontal direction and 5.74 Hz in the vertical direction. The natural frequencies of the cooling tower are low with respect to typical frequencies of Category I structures in nuclear power plants, but they agree with the characteristics of hi-rise buildings.

The circumferential distribution of the wind pressure is approximated by a 8-term Fourier cosine series including an internal wind suction of 0.5. The deflected shape of the cooling tower under the critical load combination "0.9Dead + 1.3Wind" is shown in Figure 2.

Dynamic time history analyses are performed to represent the complex interplay of the dynamic characteristics of the cooling tower and the input wind-pressure excitation in terms of gust factors. The gust factors are found to be 1.32 and 1.20 for the basic wind pressure and tornado, respectively.

The vertical distribution of the meridional (vertical) stress resultant along the windward meridian is displayed in Figure 4. Analysis results indicate that the two cooling towers will not collapse and will not experience any significant damage when subjected to the design basis earthquake or a 90-mph wind storm. However, the two cooling towers are expected to collapse if subjected in a direct hit to a 300-mph tornado.

The nonlinear finite element analyses including base uplift performed for this study and the literature research on past failures of cooling towers due to severe wind storms confirm that the mode of failure will not be the overturning cantilever tree-type and the towers will collapse inwardly with the exception of few isolated debris. The nonlinear analysis indicates a continuous increase in the lateral displacement of the shell at the throat location till excessive displacements are displayed and shell buckling occurs.

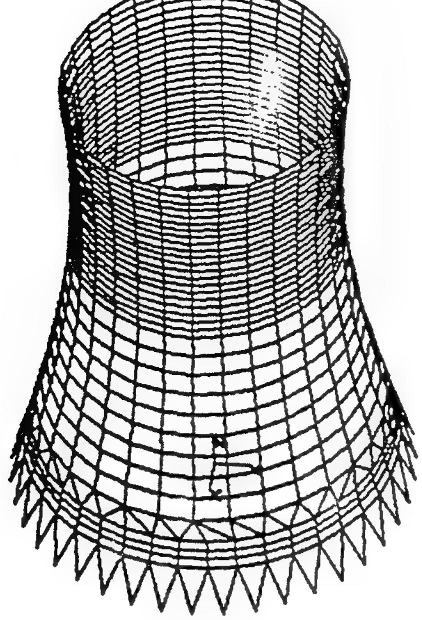

Figure 1 Finite Element Model of Cooling Tower

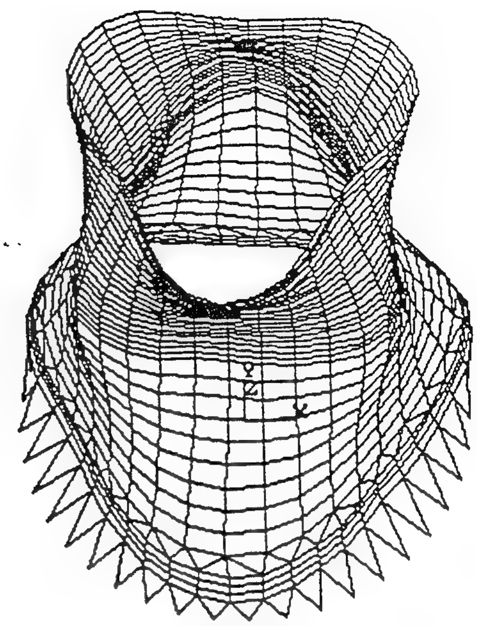

Figure 3 First Mode Shape

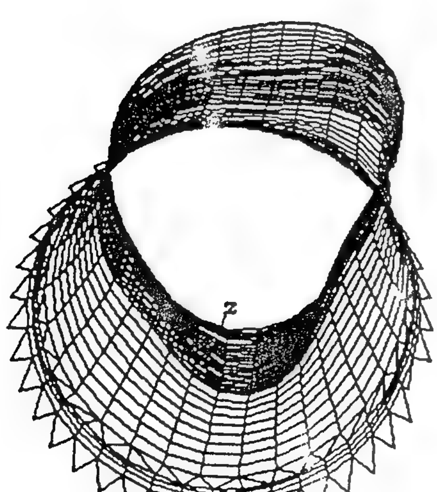

Figure 2 Deflected Shape of Cooling Tower under "0.9Dead + 1.3Wind"

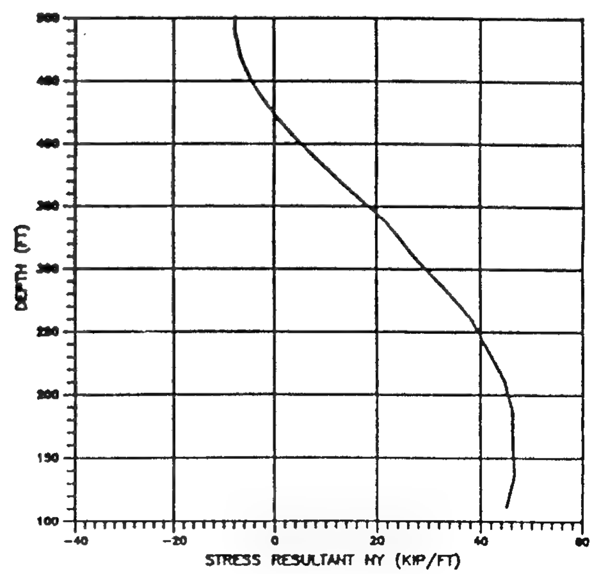

Figure 4 Vertical Distribution of Meridional Stress Resultant along Windward Meridian

3D FINITE ELEMENT MODELING AND ANALYSIS OF ARMORED VEHICLE HULLS WITH MULTIPLE ACCESS OPENINGS

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ABSTRACT

Light combat vehicles and armored personnel carriers are being postulated as playing an increasingly important support role for both troops and other more heavily armored combat vehicles. As such, they have a much greater risk than in previous roles of being subjected to severe battlefield conditions. Accuracy of determination of dynamic response of these vehicles is directly dependent on the degree of refinement of the generated model and how well the model incorporates the essential features of the vehicle and correlates to its important characteristics without being overburdened by nonessential details. Additionally, nonlinear components of vehicles with local oscillation modes can have significant influence on the global response of the vehicle and could be bracketed using a basic model and a tight-fitting configuration. As a result, driver and cargo hatch as well as engine access and rear door openings must be included in the finite element model of the basic hull to allow fair comparison and validation with experiments. The tight-fitting components such as the hatch covers and doors may have to be modeled with appropriate boundary conditions for the refined model. The current effort is devoted to the development of finite element models of the armored vehicle hull structures to facilitate simulation of low frequency vibrational response characteristics.

In this study, a bare hull of the M113 Armored Personnel Carrier with driver's, commander's and cargo hatch cutouts as well as rear door, engine access openings were carefully incorporated in the basic model which was developed using the PATRAN 3 model generator and translated into the ADINA input model with appropriate material properties and varying thicknesses of the front, rear, roof, floor and right and left side walls of the vehicle. The aluminum floor braces towards the rear of the crew compartment were incorporated as a seperate group. The patch model was updated to include two engine exhaust grill openings on the top roof towards the inclined front upper glacis. Additionally, the 3D finite element model was checked to assure continuity of element rows on adjacent normal surfaces.

The finite element model was represented by 1233 quadrilateral shell elements and 1294 nodes subdivided into seven groups based on material properties and thicknesses. All midsurface nodes were allowed to move freely along 5 degrees-of-freedom with one rotational degree-of- freedom restrained while nodes along free edges and corners were left unrestrained along all six translational and rotational degrees-of-freedom. No external forcing functions were necessary in the model to excite the vi-

brational modes for this phase of the dynamic simulation.

The updated model was run as a linear dynamic model using the ADINA finite element code to generate the first 25 natural frequencies of vibration and the corresponding modeshapes using the Bathe subspace iteration method. Ignoring the first six approximately zero eigenfrequencies which correlates to 6 rigid body vibrational modes due to removal of restraints at the points of support for the bottom floor, the frequencies ranged from a lowest frequency of 33.72 cycles/sec for the firat natural to 141.3 cycles/sec corresponding to the 20th natural frequency. Initial vibrational modes indicated torsional modes followed by bending modes of deformation of the top roof and side wall surfaces. A magnification factor of 5.0 was used for postprocessing of modeshape plots using the adinaplot subroutine. Isometric views depicting deformation of both top and bottom surfaces were generated for the first 25 eigenfrequencies. The addition of floor braces did not alter the frequency more than .5 cycles/sec.

The eigenfrequencies of the bare metallic hull were compared with those for the continuous hull to assess the influence of cutouts and access openings on vibrational response. The inclusion of multiple cutouts resulted in a noticeable reduction in weight and corresponding mass of the hull without significant reduction in stiffness. As a result, the first natural frequency which is inversely proportional to the square root of the mass and directly related to the square root of the stiffness of the vehicle was significantly higher for the refined cutout model in comparison with the crude continuous model whose lowest natural frequency was only 24.41 cycles/sec.

The refined cutout model was rerun in ADINA using orthotropic elastic material properties of the S-2 glass polyester (GRP) woven lightweight composite laminate properties with 9 material constants and variable thicknesses for each side wall with the exception of elastic properties of aluminum included for the bottom floor braces. Excluding the rigid body modes, the natural frequencies for the identical composite vehicle hull were significantly lower than those for the corresponding metallic hull. The first natural frequency for the composite hull was only 19.38 cycles/sec compared to 33.72 cycles/sec for the metallic hull. This may not be totally unexpected since the reduction of 33% of the total weight of the composite hull has been accompanied by a substantial decrease in stiffness as indicated by the significantly lower orthotropic linear elastic material moduli of GRP resulting in the computed reduction of natural frequencies.

Wednesday, June 15

1530-1710

Session 14. Localization and Normal Modes

Modal Analysis for Non-Linear Structural Systems.

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ABSTRACT

In the past decades, a lot of effort has been put into the study of the behavior of non-linear systems, both qualitatively and numerically. However, due to the fact that superposition does not hold for these systems, no satisfactory method has been developed to determine their response to an arbitrary excitation in a way that would mimic linear modal analysis for linear systems. Therefore, when such a response is needed, a linear modal analysis of the non-linear system is typically performed, and the resulting set of non-linear equations is truncated to retain a small number of linear modes —typically one or two. The resulting reduced-order model may be inaccurate —or even qualitatively incorrect, as in the case of internal resonances— due to the loss of the non-linear interactions between the modeled and unmodeled linear modes, but increasing its size to include additional linear modes can yield a computationally expensive model.

Many relevant ideas have been generalized from linear systems to non-linear systems. The recent definition of non-linear normal modes of vibration as motions occurring on invariant manifolds allows one to incorporate the effects of several linear normal modes into one so-called non-linear normal mode. The invariance property ensures that any motion starting exactly in one non-linear normal mode will be comprised only of that one for all time and will not generate any motion in the other non-linear normal modes. This is very suitable for a restricted class of motions —namely, those lying on the manifolds characterizing the non-linear normal modes— or if a single-mode model of the system is needed.

However, just like the primary use of normal modes of motion of linear systems is the modal analysis associated to them, the concept of non-linear normal modes suggests the definition of a proper "non-linear modal analysis" in order to be able to obtain the response of a system under general excitation in terms of some non-linear modal coordinates. Moreover, one ought to be able to perform model reductions using the non-linear modal coordinates —as is done for linear systems—, which requires the development of efficient truncation procedures, the ultimate goal being to be able to use fewer non-linear modes than linear ones to perform equally accurate modal analyses of non-linear systems.

Given the definition of the non-linear normal modes in terms of two-dimensional invariant manifolds, it is clear that (1) they will not interact during a pure modal motion, and (2) they are bound to interact during more general motions. Therefore, in order to extend modal analysis ideas to non-linear systems, it is essential to be able to account for the interactions between the various non-linear modes involved in the dynamics of the particular system at hand. Unfortunately, not only are those not readily available with the current formulation, but it is believed at this point that, even if proper interac-

tions between the individually invariant non-linear modes could be recovered, the non-modeled non-linear normal modes would certainly be contaminated by this process, which might not allow for reliable low-order models.

Consequently, a new formulation has been developed to ensure the invariance of the set of modeled non-linear modes with respect to the non-modeled ones, which essentially generalizes the individually invariant non-linear normal mode manifolds to multi-mode invariant manifolds. A multi-mode manifold is of dimension $2M$ when M non-linear modes are modeled, and includes the influence of all of the M individual non-linear manifolds defined previously. Besides, the interactions between the various modeled non-linear modes are accounted for at the very first stage of the definition process, thus eliminating the need for later work to recover them. The generation of a multi-mode invariant manifold follows very closely that of an individually invariant manifold, and approximations for weakly non-linear systems can be constructed easily using the same method. In the same manner as individually invariant non-linear modes do not interact during pure modal motions, the modes constituting a multi-mode manifold do not interact with the non-modeled ones for motions occurring on that manifold, hence ensuring non-contamination of the non-modeled modes if all relevant modes are embedded in the multi-mode manifold to begin with.

Numerical results obtained by the latter formulation illustrate its benefits compared to classical linear modal analyses of non-linear systems (*i.e.*, projections of equations of motion onto the linear modes). In general, the dynamics recovered by the multi-mode manifold methodology is more accurate than those obtained by a linear modal analysis using the same number of linear modes, since the multi-mode manifold reduces to this linear subspace upon linearization. In the worst case (*i.e.*, in the case of linear systems), the results are identical, while they might be much improved when the non-linearities increase. The computational savings thus obtained will of course be case-dependent, but are expected to be significant.

LOCALIZED AND NON-LOCALIZED NONLINEAR NORMAL MODES IN A MULTI-SPAN BEAM WITH GEOMETRIC NONLINEARITIES

by

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ABSTRACT

The nonlinear normal modes of a geometrically nonlinear multi-span beam consisting of n segments, coupled by means of torsional stiffeners are examined. Assuming that the stiffeners possess large torsional stiffness, the beam displacements are decomposed into static and flexible components. It is shown that the static components are much smaller in magnitude than the flexible ones. A Gallerkin approximation is subsequently employed to discretize the problem, whereby the computation of the nonlinear normal modes of the multi-span beam is reduced to the study of the periodic solutions of a set of weakly coupled, weakly nonlinear ordinary differential equations. Numerous stable and unstable, localized and non-localized nonlinear normal modes of the multi-span beam are detected. Assemblies consisting of n=2, 3, and 4 beam segments are examined, and are found to possess stable, strongly localized nonlinear normal modes. These are free synchronous oscillations during which only one segment of the assembly vibrates with finite amplitude. As the number of periodic segments increases, the structure of the nonlinear normal modes becomes increasingly more complicated. In the multi-span beams examined, nonlinear mode localization is generated through two distinct mechanisms: through Pitchfork or Saddle-mode mode bifurcations, or as the limit of a continuous mode branch when a coupling parameter tends to zero.

A Numerical Method for Determining Nonlinear Normal Modes

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Abstract

This paper examines a new approach for determining the nonlinear normal modes of multiple degree of freedom systems. Unlike algebraic solutions that generally assume a solution in the form of a polynomial expansion, this method makes only the assumption of repetitive motion in numerically determining the mode shapes. The advantage of this approach is that accuracy obtained in the mode shape identification if a function only of the accuracy of the numerical integration used and not the number of terms in the power series expansion. The drawback is that invariance of the modal manifolds can not be proven.

The first step of this approach is the determination of the linear modes of the linearized system. Each nonlinear normal mode is then identified one mode at a time. The assumption is made that the trajectory of the nonlinear normal mode is close to that of the linear mode at small amplitudes. The initial states of the system are chosen such that the system is moving in or close to only one mode.

The second step is to integrate the equations of motion forward from the initial states and then integrate them backwards. If the system is moving in only one mode, then the difference between the forward integration and the backward integration should be zero if the numerical integration is accurate. The error between the forwards and backwards integrations is then minimized with respect to the initial states. The final difference between the forwards and

backwards integrations represents the final error in the nonlinear normal mode shape for this amplitude.

The third step is to repeat step two using the initial states found previously multiplied by a constant greater than one. This constant is arbitrary, but a small constant will allow a faster convergence to the solution at the new amplitude. This step is repeated until the maximum amplitude for which the mode shape is desired is obtained.

Once the mode shapes have been determined numerically for a range of amplitudes, the raw data can be used as a look-up table by interpolating the results. Alternatively, polynomials or other functions can be curve fit to the data to provide an analytical representation of the mode shapes. These functions can then be substituted into the equations of motion to determine the modal equations of motion, although at this point the frequencies of oscillation have already been determined.

The final paper will discuss the difficulties involved in this process and compare the accuracy of this method to the invariant manifold method for a specific example. Further discussion of the benefits of this method will also be made.

ON NONLINEAR NORMAL MODES OF SYSTEMS WITH INTERNAL RESONANCE

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Abstract

A complex-variable invariant-manifold approach is used to construct the normal modes of weakly nonlinear discrete systems with cubic geometric nonlinearities and either a one-to-one or a three-to-one internal resonance. The nonlinear mode shapes are assumed to be slightly curved four-dimensional manifolds tangent to the linear eigenspaces of the two modes involved in the internal resonance at the equilibrium position. The dynamics on these manifolds is governed by three first-order autonomous equations. In contrast with the case of no internal resonance, the number of nonlinear normal modes may be more than the number of linear normal modes. Bifurcations of the calculated nonlinear normal modes are investigated.

DYNAMICS OF A MONO-COUPLED ELASTIC PERIODIC SYSTEM WITH MATERIAL NONLINEARITIES

by
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ABSTRACT

Standing and travelling waves are analyzed in an infinite system consisting of elastic layers with material nonlinearities, coupled by means of linear stiffnesses. In direct analogy to linear theory, the nonlinear periodic system possesses attenuation and propagation zones (AZs and PZs) in the frequency domain. Responses in AZs are synchronous motions with spatially attenuating or expanding envelopes, and are analyzed by extending an analytic methodology previously developed by the authors to study "nonlinear normal modes" in bounded nonlinear elastic structures. Nonlinear "propagation constants" are defined, and computer algebra is used to investigate the amplitude distributions of the standing waves. Motions in PZs are travelling waves, which are non-synchronous oscillations. Travelling waves are analyzed by applying the method of multiple-scales in space and time. Numerical computations are carried out to complement the analytical findings. The analytical and numerical methodologies developed in this work can be applied to the study of waves in a general class of nonlinear mono-coupled periodic systems, and can be extended to investigate waves in periodic systems with more than one coupling coordinates.

Thursday, June 16

0830-1010

Session 15. Structural Dynamics II

Statistical Properties of Nonlinear Vibrations of Elastic Beams

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The driving idea of this paper is that there are some general relationships among dynamical characteristics of vibrating systems. We arrive at this idea when we compare the systems studied in statistical mechanics and structural mechanics. One of the favorite systems of statistical mechanics is a set of mass particles connected by springs. The same kind of systems are obtained if one discretizes the equations of elastic structures.

It seems natural to assume that the laws of statistical mechanics might be valid for structural vibrations. We study the validity of this assumption for longitudinal vibrations of elastic beams by means of numerical simulations.

The major results are as follows:

1. There are a number of energy thresholds characterizing some changes in dynamical behavior. The number of thresholds is equal to the number of energy modes. Low energy threshold E_c is the value of the energy below which the temperature distribution depends on initial form of the first mode excitation. If the energy E is greater than E_c, temperatures of all nodes are equal. Similarly, E_2 is the energy threshold for the initially excited second mode, and so on, where $E_2 > E_c$. Upper energy threshold E^c corresponds to reaching of equipartition for the highest mode excitation.

2. For $E > E^c$ the laws of statistical mechanics hold.

3. For $E_c < E < E^c$ the spectrum distributions are very reminiscent of Planck's spectra, and can be fitted quite well by Planck distribution.

Thermodynamics of Chaotic Structural Dynamic Systems

by

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Most of the reported studies in the area of chaotic vibrations in structural dynamic systems are mainly concerned with nonlinear dissipative systems. Both ordinary differential equations approximation and partial differential equations formulation to the corresponding distributed parameter systems have been used in reported studies. Some of the reported studies have also considered the use of fractals and associated fractal dimensions to understand the chaotic vibration of beams.

In the field of theoretical physics, a clearer understanding of nonlinear dynamics, was possible by a careful examination of Hamiltonian systems. A specific case which opened a path for many valuable contributions is the Fermi, Pasta and Ulam paradox and the associated Hamiltonian systems. In this study, Fermi wanted to examine the effects of small nonlinear terms in a predominantly linear system of m particles interconnected by springs. Springs provided both linear and nonlinear effects. Fermi's objective was to examine if small nonlinear terms in a predominantly linear field would provide a proof of Boltzmann's hypothesis. Fermi and his co-authors work did not provide this proof. However it stimulated a significant amount of research during the following five decades.

Thus, motivated by developments in the field of theoretical physics, the objective of this paper is to report results of our studies of Hamiltonian nonlinear structural dynamic systems. As we know, we have gained a significant amount of insight in the field of structural dynamic systems by studying undamped free vibrations even though all structural dynamic systems have inherent damping. In particular, modal interaction, equipartition of energy and the existence of a quantity analogous to temperature are examined. We have considered examples of elastic rods, buckled beams and finite deformations of cantilever beams.

Dynamics of an Elastic Rod in a Fluid Pumping System

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A rod pumping system, as used to lift oil to the surface in nonflowing wells, is analyzed by describing longitudinal rod stretch vibrations as a sum of fixed–free modes. The rod oscillates vertically, driven at the fixed end by a constant speed motor through a four bar mechanism. Equilibrium is used to derive the following partial differential equation of rod upstroke and downstroke motion:

$$m\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}-EA\frac{\partial^2 u}{\partial x^2} = mg-m\ddot{y}-c\dot{y} \tag{1}$$

Where u(x,t) is rod stretch as a function of distance x from top of the rod string and time, A is cross sectional area of the rod string, E is Young's Modulus, g is the acceleration of gravity, m is mass of rod per unit length, and c is damping coefficient per unit length induced by viscous fluid forces. y, the vertical displacement of top of rod string, is a prescribed function that is derived from the four bar kinematics.

Pumping is governed by one set of boundary conditions during the upstroke and another set during the downstroke. When the bottom of the rod string is moving downward, the fluid load on the downhole pump is released and does not load the bottom of the rods, leading to the following set of boundary conditions during downstroke:

$$\begin{aligned} &u(0,t)=0 \\ &EA\frac{\partial u}{\partial x}(L,t)=0 \end{aligned} \tag{2}$$

which are active when

$$\dot{y} + \frac{\partial u}{\partial t}(L,t) > 0 \tag{3}$$

During the upstroke, the PDE in equation (1) remains unchanged and the new set of boundary conditions are

$$\begin{aligned} &u(0,t)=0 \\ &EA\frac{\partial u}{\partial x}(L,t)=Mg \end{aligned} \tag{4}$$

which are active when

$$\dot{y} + \frac{\partial u}{\partial t}(L,t) \le 0 \tag{5}$$

The latter boundary condition includes the weight of the fluid column, Mg, on the bottom

of the rod string. A transform methods is introduced here in order to handle this inhomogeneous boundary condition. We also assume a solution of the form:

$$u(x,t) = \sum_{i=1}^{n} f_i(t)\,\phi_i(x) \tag{6}$$

Further studies concentrated on the following one mode representation for both downstroke and upstroke, respectively:

$$\ddot{f}_1+\frac{c}{m}\dot{f}_1+\omega_1^2 f_1 = \begin{bmatrix} 0, & \dot{f}_1>\sqrt{\dfrac{mL}{2}}\dot{y} \\ Mg\sqrt{\dfrac{2}{mL}}, & \dot{f}_1\le\sqrt{\dfrac{mL}{2}}\dot{y} \end{bmatrix} + (mg+m\ddot{y}+c\dot{y})\sqrt{\frac{2}{mL}}\frac{2L}{\pi} \tag{7}$$

These equations are nonlinear (piecewise linear) because the presence or absence of a nonzero term on the right of equation (7) is nonlinearly dependent on velocity.

Based on response studies, a one mode representation is found to capture most of the rod string stretch at practical operating speeds, and was used to investigate response with dimensional and nondimensionalized equations at various crank speeds, crank lengths, and damping rates. For example, for speeds below 1/2 the rod first natural frequency larger viscous damping induced larger response; however this trend eventually reversed at higher crank speeds beyond 1/2 the first natural frequency. Highly damped systems appear to exhibit little amplitude change with speed, whereas lightly damped systems drastically increase in amplitude as the crank speeds approached the first natural frequency. Work on this problem is continuing, and we hope to present some preliminary results on a study of a cracked rod string.

On the quasi-steady analysis of one-degree-of-freedom galloping with combined translational and rotational effects

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ABSTRACT

Galloping is the self-excited oscillation of an elastic structure in a wind field. It occurs at the structure's natural frequency, which when sufficiently low allows a quasi-steady aerodynamic analysis, where the instantaneous wind forces are derived from an equivalent steady flow situation. For bluff body shapes normally encountered in practical galloping situations (*e.g.* power transmission cables, bridge decks, buildings) these steady-aerodynamic data are derived from static wind tunnel tests. For the two-dimensional galloping of a rigid prismatic beam in a steady, homogeneous wind field normal to its axis, the general motion is composed of a translation and a rotation of the cross-section.

The modelling of *translation* galloping is straightforward. Each point of the cross-section has the same translation vector, and an identical steady flow situation can be defined, where the effective, homogeneous wind field is defined by the vectorial difference of flow and translation velocity. In most practical situations only cross-wind galloping (also referred to as vertical or plunge galloping) is relevant, where the oscillation direction is normal to the wind vector. This one-degree-of-freedom case was considered by Parkinson [1], who solved the problem by the two-time-scale method for a square cross-section. His model provides an understanding of the fundamental aspects of this type of galloping, such as the variation of galloping amplitude with wind speed, the influence of structural damping, and the possibility of multiple limit-cycles.

For *rotational* motions the modelling is more problematic. Not only do the aerodynamic forces depend on both the rotation angle and the rotation velocity (whereas for translation the translation distance is not relevant), it is further not possible to define an exactly equivalent static situation, as each point of the cross-section has a different translation vector. Attempts have been made to model the rotation by using a "characteristic" relative velocity to approximate the averaged flow field [2]. However, this modelling has been used in situations where the rotation axis (nearly) coincides with the beam axis, where the assumption is highly questionable [3], and also often in a two-degree-of-freedom system in combination with translation. An understanding of the fundamental influence of rotation is therefore greatly inhibited by the increased complexity of the system and the suspiciousness of the quasi-steady modelling.

In this paper we present an oscillator structure which displays a rotational oscillation mode that can be modelled reliably with the quasi-steady theory. This makes it meaningful to attempt a comparison between theory and an experimental set-up. The oscillator is composed of a prismatic beam hinged around an axis parallel to the beam axis, where the arm length is large in comparison to the diameter of the beam (Fig.1). The oscillator provides the simplicity of one degree-of-freedom, while displaying a number of features essential to rotation. In approximation (for small oscillation angles), it can be understood as an extension of Parkinson's model by adding a rotation that is linearly coupled to the translation. We regard it as an intermediate step between pure translational galloping and complete two-degree-of-freedom galloping with uncoupled translation and rotation.

The analysis reveals that the aerodynamic damping is completely determined by the model's section characteristics and can be expressed in terms of an aerodynamic amplitude function. This result can be used to analyse both types of galloping, which differ only by the way the structural amplitude (displacement) is related to the aerodynamic amplitude. An interesting result is that, in contrast to translation where the limit-cycle amplitude increases linearly with the wind speed, the rotational galloping shows an aerodynamic limit for large wind speeds. A practical application may perhaps be obtained from this natural self-limiting of the galloping amplitude.

Parallel to the theoretical work an experimental set-up of the type shown in Fig.1 (with a natural frequency of about 0.5 Hz) is being developed, for validation of the analytical modelling with experiments in a wind tunnel. Preliminary measurements seem to confirm the aerodynamic limiting effect, as shown in Fig.2, where the limit-cycle amplitude is given as a function of wind speed.

References:

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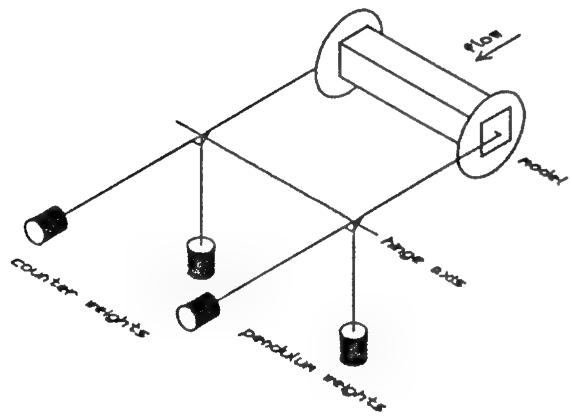

Fig.1: Schematic lay-out of the oscillator used in the study of one-degree-of-freedom galloping with combined translational and rotational effects.

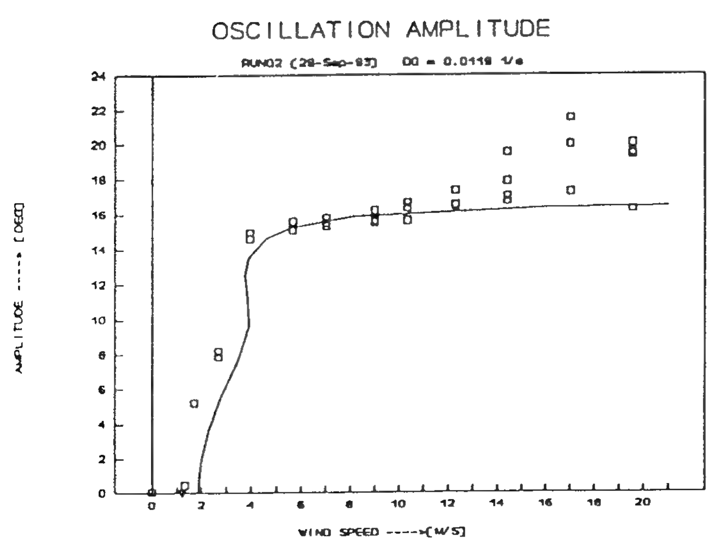

Fig.2: "Limit-cycle" amplitudes as a function of wind speed (the scatter indicates unsteady variations of the amplitude).

An Experimental Investigation of Energy Transfer from a High-Frequency Mode to a Low-Frequency Mode in a Flexible Structure

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Abstract

The objective of the present paper is to experimentally observe and characterize the transfer of energy from low-amplitude, high-frequency modes to high-amplitude, low-frequency modes. The subject of the study is a three-beam-frame structure. The excitation amplitude is restricted to below 2 g peak. We have focused on observing, characterizing, and documenting the excitation of the first mode by high-frequency forcing. The energy-transfer processes are identified by power spectra and further characterized by frequency and amplitude sweeps. The energy-transfer routes observed in the experiment are subharmonic resonance of order one-half, combination resonance of the additive type, and interaction between widely spaced modes. In the latter route, an excitation at a frequency that is more than 100 times the first-mode frequency has been observed to excite the first mode.

Thursday, June 16

1030-1210

Session 16. Structural Dynamics III

Experimental and Analytical Investigations of the Nonlinear Response of a Cantilever under Transverse Excitation

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An in-depth experimental investigation of one-to-one nonlinear coupling phenomena in the dynamic response of cantilever beams excited by a periodic transverse displacement is presented. The results of these experiments are in excellent agreement with the analytical predictions presented in other work by this author. The experiments were conducted on four aluminum beams having the following dimensions :

beam 1: $45'' \times 1/4'' \times 1/4''$; beam 2: $70.5'' \times 1/4'' \times 1/4''$

beam 3: $44'' \times 3/4'' \times 1/4''$; beam 4: $70.5'' \times 1/2'' \times 1/4''$

Beams 1 and 2 were chosen to have a nominally square cross section in order to illustrate the one-to-one nonlinear modal coupling. Beams 3 and 4 were chosen in order to investigate planar responses (v_e versus Ω) since they will not exhibit the one-to-one nonlinear response exhibited by beams 1 and 2. The layout of the experimental set-up is shown in Fig. 1. Signal generation and monitoring were accomplished using an HP 3562A Dynamic Signal Analyzer. The signals from the analyzer were amplified and sent to a long stroke electrodynamic table shaker whose base was securely clamped to a steel test-bed fixed to the floor of the room in order to eliminate any rocking or translation of the unit. A custom designed aluminum clamp assembly was used to provide a fixed support for the beams and to act as an interface between the beams and the shaker. Base excitation was chosen as a means to provide a transverse excitation to the beams without influencing their ability to move freely in three-dimensional space and thus interfere with the observation of a coupled response involving planar and nonplanar motion.

A common method of exciting flexible structures involves the use of a thin wire-like stinger attached to a shaker armature on one end and to some point on the structure on the other end. This method may be acceptable for investigating planar response of beams, but was avoided since it would clearly influence, and most probably restrict, any motion which does not coincide with the direction of the stinger stroke. The motion of the beam and the base assembly was measured using three very light piezoelectric accelerometers. The choice of accelerometers over strain gauges was made because of the simple relationship that exists between acceleration and deflection, and because of the mounting versatility provided by accelerometers. The damping coefficient of each beam was determined experimentally and then used to generate the amplitude-frequency response curves obtained from a perturbation analysis of the nonlinear integro-partial differential equations of motion for the beam. The actual response amplitude as a function of the excitation frequency was measured in the laboratory and the results were then compared with those obtained from the analysis. Excellent correlation between the experimental and the analytical results was obtained when effects such as "base flexibility" in the shaker

and nonlinear damping were accounted for in the equations of motion. Figure 2 shows the experimental and analytical frequency response of the third mode of beam 2 for a base displacement of $e^* = 0.15\,mm$ and a normalized damping coefficient $c = 0.11$ that was measured for that beam. The experimental and analytical amplitude frequency responses of the third mode for beam 4 with $e^* = 0.1\,mm$, $c = 0.1$ and three values of the nonlinear damping coefficient ($c_0 = 0, c_0 = 0.2$ and $c_0 = 0.6$) are shown in Fig. 3.

The work presented here discloses the importance of accounting for effects such as shaker imperfections and nonlinear damping in the system.

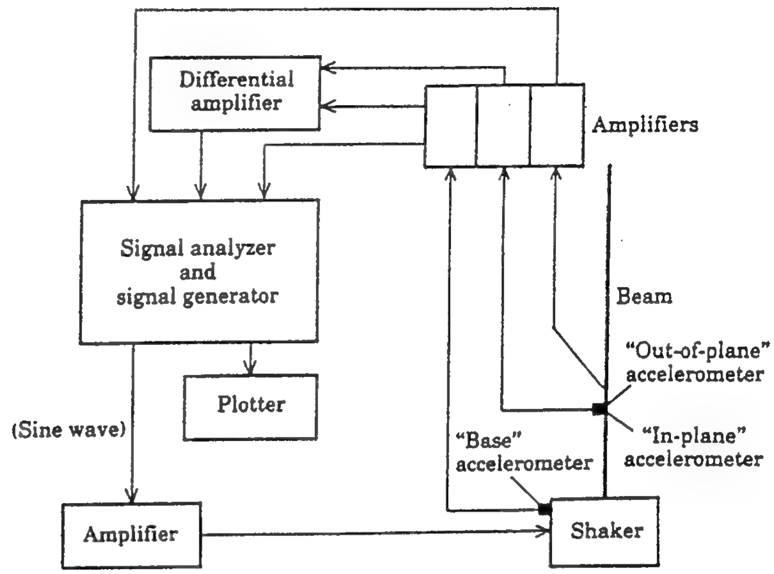

Fig.1

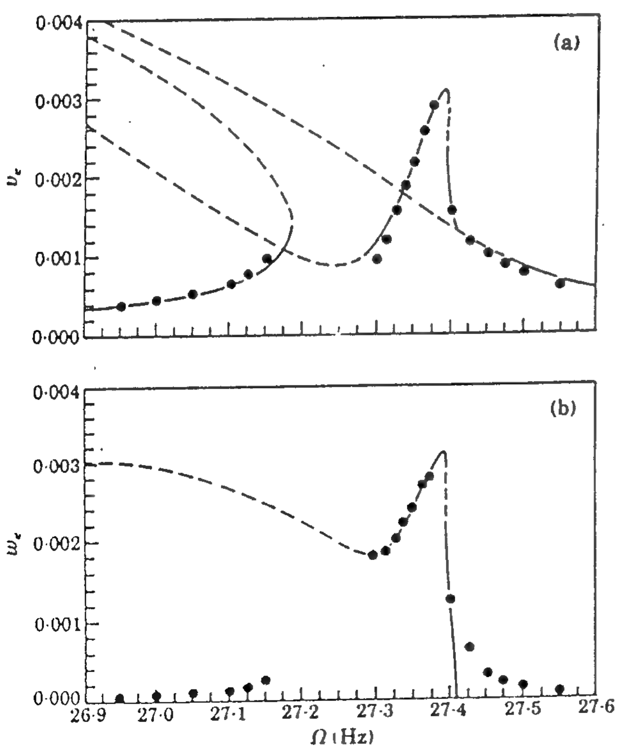

Fig.2

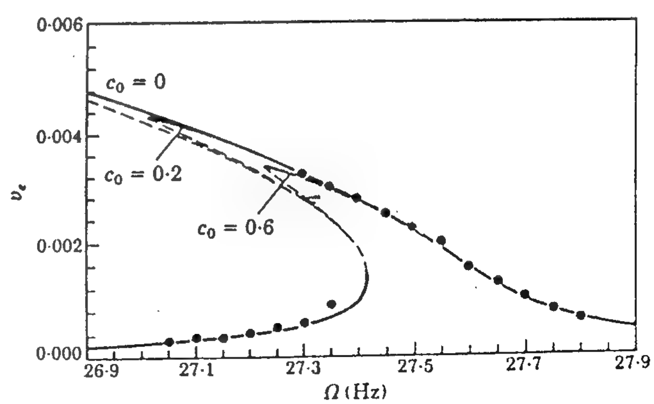

Fig.3

DYNAMICS OF A MULTI-DOF BEAM SYSTEM WITH DISCONTINUOUS SUPPORT

by

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Abstract

This paper deals with the long term behaviour of periodically excited mechanical systems consisting of linear components with many degrees of freedom and local nonlinearities. The particular system investigated is a 2D pinned-pinned beam, which halfway its length is supported by a one-sided linear spring and excited by a periodic transversal force.

The linear part of this system is modelled by means of the finite element method and subsequently reduced using a Component Mode Synthesis method.

Models with 1 and 4 degrees of freedom are investigated. Periodic solutions are computed by solving a two-point boundary value problem using finite differences. Branches of periodic solutions are followed at a changing design variable by applying a path following technique. Floquet multipliers are calculated to determine the local stability of these solutions and to identify local bifurcation points. The long term behaviour is also investigated by means of standard numerical time integration, in particular for determining chaotic motions.

In additon, the Cell Mapping technique is applied to identify period and chaotic solutions and their basins of attraction. A dedicated extension of the existing Cell Mapping methods enables to investigate systems with many degrees of freedom.

By means of the above methods very rich, complex dynamic behaviour is demonstrated for the beam system with one-sided spring support. This behaviour is confirmed by experimental results.

Nonlinear Vibrations in Beams and Frames: The Effect of The Deformed Equilibrium State

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Abstract

In the study of the nonlinear vibrations of planar frames and beams with infinitesimal displacements and strains, it is common to disregard the influence of the static displacements due to the gravity effect and other conservative loads. In this study two planar structures with a two-to-one internal resonance to a primary resonance of the second mode are analyzed with the purpose of discussing the effect of the deformed equilibrium configuration on the nonlinear vibrations.

The adopted equations of motion were derived accordingly to Mazzilli [1] and André [2]. They are explicitly defined in function of generalized displacements and their first and second time derivatives. The reduction to two degrees of freedom was done as in André [2], considering a displacement field obtained by superposing the equilibrium displacement field (resulting from a nonlinear static analysis) and the displacement field resulting from a linear combination of the selected natural modes, about the equilibrium configuration. Subsidiary conditions, defined from geometrical nonlinear equations for planar beams, are introduced to avoid losing inertial and geometrical nonlinear terms of the reduced equations of motion.

In order to provide some comparisons, two structures that have been extensively studied by several researchers are selected for analysis. Initially, the L-beam with homogeneous material and two discrete masses is analyzed. This beam was previously studied by Haddow, Barr and Mook [3], Nayfeh and Zavodney [4], Nayfeh, Balachandran, Colbert and Nayfeh [5], Nayfeh and Balachandran [6] and André [2]. It is also analyzed a portal frame with three discrete masses, formerly studied by Barr and McWhannel [7], Mazzilli [1], André and Crespo da Silva [8] and André [9].

In the two cases analyzed it was verified that the deformed equilibrium configuration virtually coincides with the undeformed configuration. The first two lower frequencies do not present differences larger than 2%, noting that this difference is bigger in the first mode, and the modes are practically indistinguishable for the deformed and undeformed configuration.

The frequency response curves clearly show that the effect of the deformed equilibrium configuration results in a significant translation along the detuning factor axis. Moreover, the consequence in the amplitude response curves is, obviously, even more important, considering that the phenomenon represented by the curves would be distinct for the same value of the detuning factor.

These results show that the static displacements that define the deformed equilibrium configuration may have a strong influence on the nonlinear vibration responses of a structure. This fact reveals the importance of considering such effect, or, at least, the importance of studying carefully the sensibility of each particular structure to geometrical imperfections. Disregarding this effect may lead to obtaining inaccurate results, mainly if one considers that certain kinds of answers occur in narrow ranges of the detuning factor.

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Nonlinear Dynamics of a Cantilever Beam carrying a Moving Mass

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Abstract

In this paper the dynamic behaviour of a flexible beam carrying a moving mass is studied using the finite element formulation which incorporates an adaptive mass matrix. The mass matrix represents the total mass distribution in the *system*, and accounts for the nonlinearities resulting from the motion of the mass.

The mathematical model of the system proposed by F.Khalily [1,2] included two coupled nonlinear integral/partial differential equations which were impossible to solve analytically and difficult to solve numerically in their existing form. In her approach the partial differential equations were reduced to a set of ordinary differential equations by assuming a trial solution which was separable in space and time and also 'improved shape functions' were developed, which took into account the position of the mass. Moreover the Ritz technique used in [1,2] was an approximate method which was valid for small amplitude oscillations of the beam, and was only applicable to small masses. As a remedy a finite element formulation is employed in such a way that an adaptive mass matrix, which transforms as the mass traverses the beam, results. This version of a finite element solution eliminates the use of 'improved shape functions' developed by F.Khalily, is easier to apply and is more robust. The model is more accurate as the formulation includes all possible nonlinearities in the *system*, and is also valid for large amplitude oscillations of the beam. The new method works equally well for load masses of any size, small or large.

Case Studies have been conducted for small and large payloads to affirm the validity of the finite element code. Four special test cases have been studied, namely: 1) Mass fixed to the middle of the beam, 2) Mass fixed to the end of the beam, 3) a mass with its motion prescribed, and finally 4) beam - mass interaction due to nonlinear coupling were compared with the theory and results of [1,2] for a small mass. The close agreement of the results from the above methods validates the finite element model and illustrates the soundness of the mathematical model presented in [1,2].

In the latter test case the nonlinear 'Internal Resonance' phenomenon was observed in which the nonlinear coupling between the beam and the mass generated an energy link, thereby facilitating a transfer of oscillatory energy between the modes of vibration of the beam and the mass. A damper was added to the mass and an optimum damping ratio administered to keep the beam oscillations minimal.

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Vibrations of a Portal Frame Excited by a Non-Ideal Motor

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Structures supporting unbalanced machines capable of a limited power output are considered, as in the case of real motors. The motion of an oscillating structure under the action of such an energy source is accompanied by interaction between these non-ideal motors and their support. In usual approaches, the excitation is considered as ideal; that is, the influence of the motion of the structure on the motor is disregarded. Here, the reciprocal influence of the system on the energy source is considered. As consequences, in the regions of resonance unstable conditions of motion occur, the form of resonance curve depends on which direction the frequency of the excitation is being altered, the form of the oscillations is changed, and the character of the transition of the system through resonances is altered.

As an application, the simple portal frame in the Fig.1a is analyzed. It has two columns clamped at their bases with height h and constant cross section with concentrated weights at their tops of mass m. The horizontal beam is pinned to the columns at both ends with length L and constant cross section. Linear elastic material is considered. The adopted model of the structure is shown in the Fig.1b. The coordinate q_1 is related to the horizontal displacement of the top of the columns in the sway mode (with natural frequency ω_1) and q_2 to the mid-span vertical displacement of the beam in the first symmetrical mode (with natural frequency ω_2). Coefficients of modal linear viscous damping c_1 and c_2 are adopted. An unbalanced non-ideal motor is placed at the mid-span of the beam. The angular displacement of its rotor is given by q_3. It has total mass M, its rotor has moment of inertia I and carries an unbalanced mass m_0 at a distance r from the axis. The characteristic driving torque of the motor $\mathcal{L}(\dot{q}_3)$ and the resisting torque $\mathcal{H}(\dot{q}_3)$, for each given power level, are assumed to be known. The resulting equations of motion are:

$$\ddot{q}_1 + \omega_1^2 q_1 = \frac{m_0 r}{(2m+M)h}(\ddot{q}_3 \sin q_3 + \dot{q}_3^2 \cos q_3) - \frac{c_1}{(2m+M)h}\dot{q}_1 \tag{1}$$

$$\ddot{q}_2 + \omega_2^2 q_2 = \frac{m_0 r}{ML}(-\ddot{q}_3 \cos q_3 + \dot{q}_3^2 \sin q_3) - \frac{c_2}{ML}\dot{q}_2 - \frac{g}{L} \tag{2}$$

$$\ddot{q}_3 = \frac{m_0 h r}{(I+r^2 m_0)}\ddot{q}_1 \sin q_3 - \frac{m_0 L r}{(I+r^2 m_0)}\ddot{q}_2 \cos q_3 + \frac{\mathcal{L}(\dot{q}_3) - \mathcal{H}(\dot{q}_3)}{(I+r^2 m_0)} \tag{3}$$

A direct numerical treatment of this system is pursued without any previous approximation. A standard Runge-Kutta 4^{th} order algorithm is used. To allow for that, the equations are algebraically manipulated and recast as:

$$\ddot{q}_1 = f_1(q_1, \dot{q}_1, q_3, \dot{q}_3) \tag{4}$$

$$\ddot{q}_2 = f_2(q_2, \dot{q}_2, q_3, \dot{q}_3) \tag{5}$$

$$\ddot{q}_3 = f_3(\dot{q}_3) \tag{6}$$

The numerical values adopted are: $EI = 128Nm^2$, $h = 0.36m$, $L = 0.5m$, $M = 2.0kg$, $m = 0.5kg$, $m_o = 0.1kg$, $I = 0.00017kgm^2$, $r = 0.01m$, $c_1 = c_2 = 31.36Ns/m$. Passage through resonance with the second natural frequency, corresponding to the mid-span vertical displacement of the beam in the first symmetrical mode ($\omega_2 = 157rad/s$), is presented. The characteristic net torque of the motor around this region is supposed to be given by a set of straight lines of the form:

$$\mathcal{L}(\dot{q}_3) - \mathcal{H}(\dot{q}_3) = a - b\dot{q}_3 \tag{7}$$

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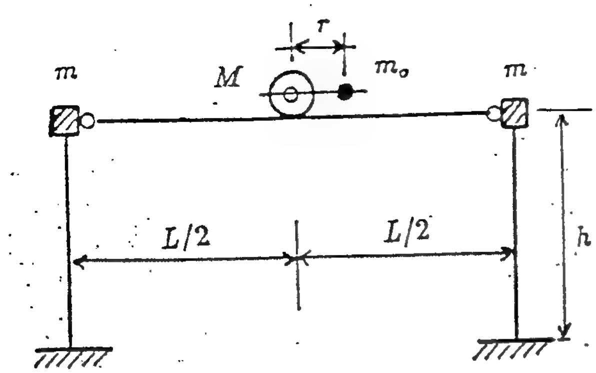

Figure 1a

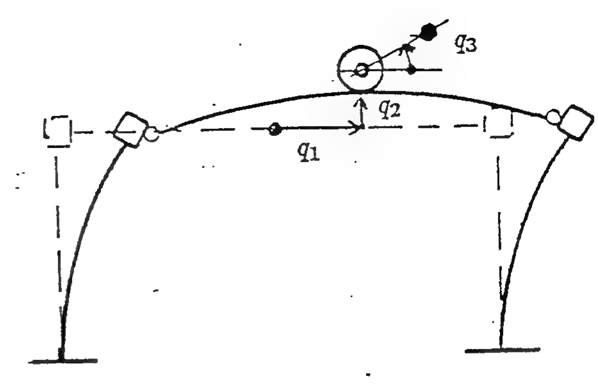

Figure 1b

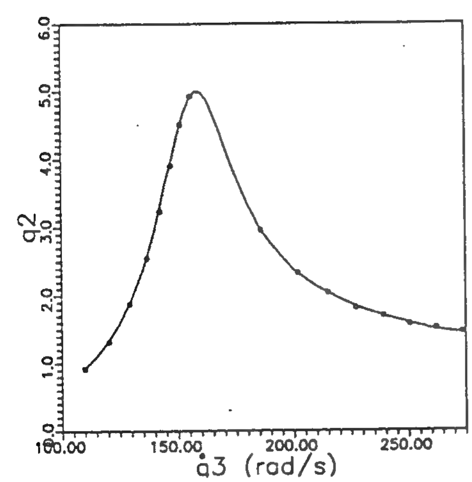

Figure 2

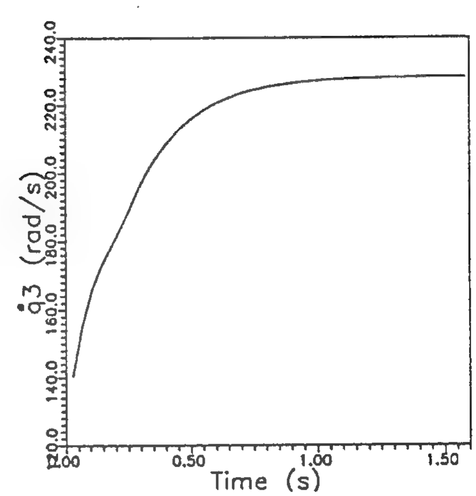

Figure 3

The solid line curve in Fig.2 shows the variation of coordinate q_2 with the angular speed of the motor $\dot{q_3}$ in the ideal case. Plots of stable motions in the non-ideal case for the several adopted characteristic lines are represented by small circles. As predicted, no stable solutions are found inside a fairly large band of frequencies to the right side of the resonance peak. As an example, for $a = 0.2Nm$ and $b = 0.001Nm/s$, a transient analysis started with a initial motor speed $\dot{q_3} = 125rad/s$ is shown in Fig.3. Stable conditions of vibration are only reached at the opposite side of the resonance peak for $\dot{q_3} = 225rad/s$.

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The Hunting And Hopf Bifurcation in A Railway Vehicle with Hysteretic Nonlinearities in Its Hunging System

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1、Intrduction

Scientists have paid much attention to the study of lateral stability of railway vehicles . Some of them have tried to use bifurcation theory to solve the problems in vehicle dynamics[1−3]. Based on the averaging method, this paper investigates a seventeen degree of freedom railway vehicle system. The Hopf bifurcation solution and its stability criteria are obtained. It is found that the stable regions exist in a saparated form in the parameter space and the bifurcation is a supercritical one. The process in this paper is very easy to be programed, so it is convenient to use it in the analyses of multiple degree of fredom nonlinear systems.

2、A Railway Vehicle System with Hysteretic Nonlinearities.

Consider a railway vehicle system with hysterestic nonlinearities in its hunging systems.The system's differential equation is [4]

$$\dot{X} = A(V)X + F_1(X,V) \qquad X \in R^{34} \tag{1}$$

where, A is a 34×34 matrix, V is the velocity.

Let $V = V_c + \mu$,where μ is a small parameter.Expand A(V) in power series of μ,and rewrite equation (1) as:

$$\dot{X} = A_0(V_c)X + F(X,\mu) \qquad X \in R^{34} \tag{2}$$

Suppose $A_0(V_c)$has only a pair of pure imaginary eigenvalues $\pm i\omega$, and the other eigenvalues have strictly negative real parts. Let the solution of equation (2) is

$$X = 2a(t)(\alpha cos\varphi - \beta sin\varphi), \qquad \varphi(t) = \omega t + \theta(t) \tag{3}$$

Following the procedure of [5], we can get the standard equation in averaging method as following.

$$\begin{cases} \dot{a} = \dfrac{1}{2} Y^T F = R \\ \dot{\theta} = \dfrac{1}{2a} Y^{*T} F = S \end{cases} \tag{4}$$

Averaging equation (4),we can get:

$$\begin{cases} \dot{a} = C_{a1} a + C_{an} a^n \\ \dot{\theta} = C_{b1} + C_{bn} a^{n-1} \end{cases} \tag{5}$$

The amplitudes of the steady state solutions are:

$$a = 0 \qquad a = \left(-\frac{C_{a1}}{C_{an}}\right)^{\frac{1}{n-1}} \tag{6}$$

The existing and stable condition is $C_{a1} > 0 \quad C_{an} < 0$ (7)

After the equilibrium point loses its stability, a periodic solution may bifurcat from it. The first order approximate of the bifurcating solution is

$$X = 2a(\alpha cos\varphi - \beta sin\varphi), \ \ \varphi(t) = \omega t + \theta \qquad (8)$$

$$a = (-\frac{C_{a1}}{C_{an}})^{\frac{1}{n-1}}; \qquad \theta = (C_{b1} - C_{bn}\frac{C_{a1}}{C_{an}})t + \theta_0$$

3、The Influence of System Parameters on The Bifurcation Solutions

The study of the influence of the system parameters on the amplitudes of the bifurcation solutions shows that

(1)、The bifurcation is a supercritical one.

(2)、There exist stable and unstable regions of the bifurcation solution in the system parameter space.These regions are not of the same size and they are saparated. When the speed of the vehicle is above its critical speed V_c, and if the bifurcation solution is stable, the running quality of the vehicle will not get worse, as long as the amplitude of the bifurcation solution is small enough. Therefore a principle is offered here which suggests that the desiging of vehicles consider not only the linear criticle speed but also the stable regions,in other words, choose the system parameters in the middle of a larger stable region, so as not to make the bofurcation solution unstable by small disturbance.

(3)、The amplitude of the bifurcation solution decreases with the increase of hysteretic coefficient η. The amplitude and the stable regions will change with the change of system parameters. The change of the amplitude is reverse proportion to the change of λ, K_{1x}、K_{1y}, and direct proportion to that of W.It is also shown that K_{1z} *and* K_{2z} have little influence on the amplitudes.

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ASYMPTOTIC METHODS IN THE THEORY OF ESSENTIALLY NONLINEAR DYNAMICS PROBLEMS

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Asymptotical approach is the natural method for the solving of nonlinear dynamical problems. As a rule, quasi linear asymptotics, is based on the assumption of small amplitude, have been used in this field. But this choice of small parameter is not optimal. In this papers were proposed a new asymptotic method, the main idea of it is confined in the introduction of small parameter in the power of nonlinear terms. and using theory of distribution. For example, Duffing equation ($\partial^2 x/\partial t^2 + x + \varepsilon x^3 = 0$) may be solved by means of an expansion in power of ε (Nayfeh) or by the introducing "small" parameter δ as follows (Bender et al.): $\partial^2 x/\partial t^2 + x + \varepsilon x^{1+\delta} = 0$. The last method is the more effective in comparison with tradition quasi linear expansions. At that time it will be very interesting to obtain asymptotical solution of this equations for $\delta \to \infty$.

V.N.Pilipchuk proposed effective method for solving essentially nonlinear dynamics problems. The main idea of this approach is based on the replacing initial system by simple

impact-vibrating one. This approach brought good results, but remained open the questions of justification and construction of high approximation. Those problems may be solved because of the application of the new asymptotic algorithm. In correspondence to it we first of all split the term x^n in the powers of parameter 1/n. The coefficients of this series contain δ Dirac function ant its derivatives. This procedure are justified in the framework of the distribution theory. Then we split initial equation to the recurrent infinite system of linear equation with distribution as coefficients and right sides, that may be solved on the basis of well produced methods.

Approach proposed above is the natural asymptotic method for solving differential equations which contain term $x^{1+\delta}$ for $\delta \to \infty$. Similar proposed asymptotical approach for the case of small δ was proposed . Matching solutions for $\delta \to 0$ and $\delta \to \infty$ by means of two-point Pade approximant one can obtain solution for any value of δ.

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